1. Find all integers \( n \) for which \( n^4 + 6n^3 + 11n^2 + 3n + 31 \) is a perfect square.

2. The planar diagram below, with equilateral triangles and regular hexagons, sides length 2cm., is folded along the dashed edges of the polygons, to create a closed surface in three dimensional Euclidean spaces. Edges on the periphery of the planar diagram are identified (or glued) with precisely one other edge on the periphery in a natural way. Thus for example, BA will be joined to QP and AC will be joined to DC. Find the volume of the three-dimensional region enclosed by the resulting surface.

![Planar diagram](image)

3. Let \( (a_i)_{1 \leq i \leq 2015} \) be a sequence consisting of 2015 integers, and let \( (k_i)_{1 \leq i \leq 2015} \) be a sequence of 2015 positive integers (positive integer excludes 0). Let

\[
A = \begin{pmatrix}
    a_1^{k_1} & a_2^{k_2} & \cdots & a_{2015}^{k_{2015}} \\
    a_1^{k_1} & a_2^{k_2} & \cdots & a_{2015}^{k_{2015}} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_1^{k_1} & a_2^{k_2} & \cdots & a_{2015}^{k_{2015}}
\end{pmatrix}.
\]

Prove that 2015! divides \( \det A \).
4. Consider the harmonic series \( \sum_{n \geq 1} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} \ldots \). Prove that every positive rational number can be obtained as an unordered partial sum of this series. (An unordered partial sum may skip some of the terms \( \frac{1}{k} \).)

5. Evaluate \( \int_0^\infty \frac{\arctan(\pi x) - \arctan(x)}{x} \, dx \) (where \( 0 \leq \arctan(x) < \pi/2 \) for \( 0 \leq x < \infty \)).

6. Let \((a_1, b_1), \ldots, (a_n, b_n)\) be \( n \) points in \( \mathbb{R}^2 \) (where \( \mathbb{R} \) denotes the real numbers), and let \( \varepsilon > 0 \) be a positive number. Can we find a real-valued function \( f(x, y) \) that satisfies the following three conditions?

   (a) \( f(0, 0) = 1 \);

   (b) \( f(x, y) \neq 0 \) for only finitely many \((x, y) \in \mathbb{R}^2\);

   (c) \( \sum_{r=1}^{n} |f(x + a_r, y + b_r) - f(x, y)| < \varepsilon \) for every \((x, y) \in \mathbb{R}^2\).

   Justify your answer.

7. Let \( n \) be a positive integer and let \( x_1, \ldots, x_n \) be \( n \) nonzero points in \( \mathbb{R}^2 \).

   Suppose \( \langle x_i, x_j \rangle \) (scalar or dot product) is a rational number for all \( i, j \) (1 \( \leq i, j \leq n \)). Let \( S \) denote all points of \( \mathbb{R}^2 \) of the form \( \sum_{i=1}^{n} a_i x_i \) where the \( a_i \) are integers. A closed disk of radius \( R \) and center \( P \) is the set of points at distance at most \( R \) from \( P \) (includes the points distance \( R \) from \( P \)). Prove that there exists a positive number \( R \) and closed disks \( D_1, D_2, \ldots \) of radius \( R \) such that

   (a) Each disk contains exactly two points of \( S \);

   (b) Every point of \( S \) lies in at least one disk;

   (c) Two distinct disks intersect in at most one point.