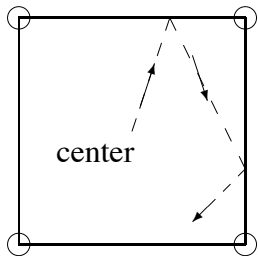


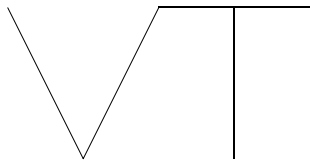
**15th Annual**  
**Virginia Tech Regional Mathematics Contest**  
From 9:00 a.m. to 11:30 a.m., October 30, 1993

**Fill out the individual registration form**

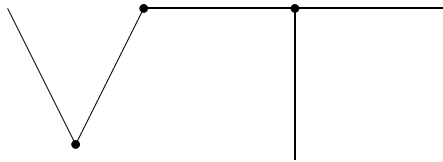
1. Prove that  $\int_0^1 \int_{x^2}^1 e^{y^{3/2}} dy dx = \frac{2e-2}{3}$ .
2. Prove that if  $f: \mathbb{R} \rightarrow \mathbb{R}$  is continuous and  $f(x) = \int_0^x f(t) dt$ , then  $f(x)$  is identically zero.
3. Let  $f_1(x) = x$  and  $f_{n+1}(x) = x^{f_n(x)}$ , for  $n = 1, 2, \dots$ . Prove that  $f'_n(1) = 1$  and  $f''_n(1) = 2$ , for all  $n \geq 2$ .
4. Prove that a triangle in the plane whose vertices have integer coordinates cannot be equilateral.
5. Find  $\sum_{n=1}^{\infty} \frac{3^{-n}}{n}$ .
6. Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a surjective map with the property that if the points  $A$ ,  $B$  and  $C$  are collinear, then so are  $f(A)$ ,  $f(B)$  and  $f(C)$ . Prove that  $f$  is bijective.
7. On a small square billiard table with sides of length 2 ft., a ball is played from the center and after rebounding off the sides several times, goes into a cup at one of the corners. Prove that the total distance travelled by the ball is **not** an integer number of feet.



8. A popular Virginia Tech logo looks something like



Suppose that wire-frame copies of this logo are constructed of 5 equal pieces of wire welded at three places as shown:



If bending is allowed, but no re-welding, show clearly how to cut the maximum possible number of ready-made copies of such a logo from the piece of welded wire mesh shown. Also, prove that no larger number is possible.

