Fourier Series Meets Linear Algebra
Part 3: Fourier series

P1) Let H_n denote the linear class of functions spanned by $1, \cos(x), \cos(2x), \ldots, \cos(nx)$, $\sin(x), \sin(2x), \ldots, \sin(nx)$.

a) What is the dimension of H_n?

b) Is H_n a subspace of H_{n+1}?

c) For what values of n is $\sin^2(x)$ an element of H_n?

P2) Show that $\frac{sin(x)}{\sqrt{\pi/2}}, \frac{sin(2x)}{\sqrt{\pi/2}}, \frac{sin(3x)}{\sqrt{\pi/2}}, \ldots$ is an orthonormal sequence on $[0, \pi]$. [Show all calculations, do not just cite results from the slides.]

P3) a) Sketch the period 2π extension of the function defined in the interval $0 \leq x < 2\pi$ by

$$f(x) = \begin{cases} 1 & \text{if } x \leq \pi \\ -1 & x > \pi \end{cases}$$

b) Find the Fourier series of $f(x)$.

c) Does the Fourier series converge to $f(x)$?

P4) a) Sketch the period $2L$ extension of the function defined in the interval $-L \leq x < L$ by

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x < L \\ -1 & \text{if } -L \leq x < 0 \end{cases}$$

b) Find the Fourier series of $f(x)$.

c) Does the Fourier series converge to $f(x)$?