Fourier Series Meets Linear Algebra
Part 2: Orthogonality

P1) Prove that, if nonzero vectors f and g are orthogonal, neither can be a scalar multiple of the other.

P2) Is $1, x, x^2, x^3, \ldots$ an orthogonal sequence in $C[0,1]$?

P3) Let f and g be elements of $C[0,1]$ defined by $f(x) = 1$ and $g(x) = x$. Find the projection of f in the direction of g.

P4) Using the Gram-Schmidt process, find an orthonormal basis for the three-dimensional subspace of $C[-1,1]$ spanned by $1, x, x^2$.

P5) Let W_n be a subspace with an orthonormal basis $\phi_1, \ldots, \phi_n$. If $g = \text{proj}_{W_n}(f)$ then what is $\text{proj}_{W_n}(g)$?