A Flexible Methodology for Optimal Helical Compression Spring Design

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Problem

- An algorithm to design any general spring with interchangeable constraints and objectives.
- Quantify and incorporate stress relaxation and creep
- Incorporation of uncertainty into design optimization.

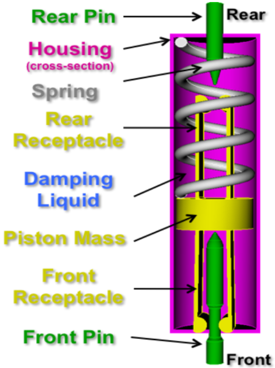


Figure : Acceleration switch

Algorithm

- What do we need to include in the algorithm?
  - Feasibility
  - Sensitivity Analysis
  - Optimization

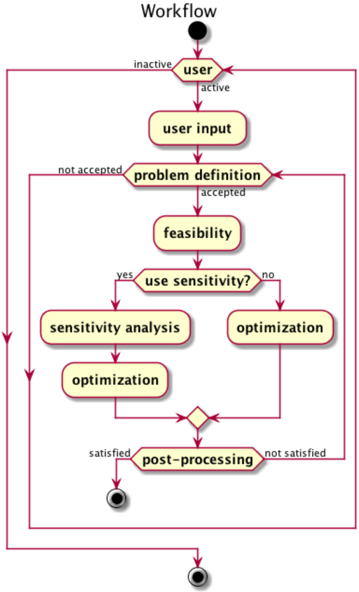
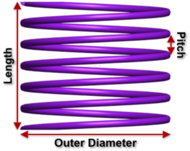
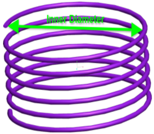
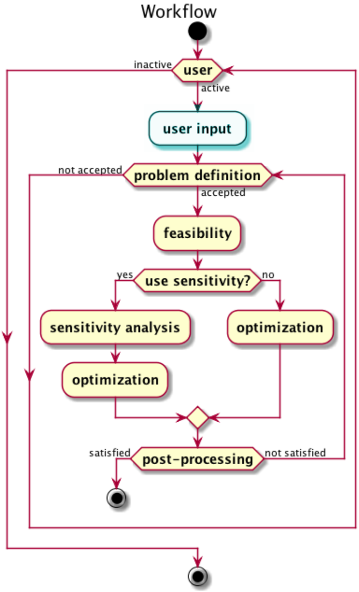


Figure : Algorithm of approach

User Input


Length
Pitch
Outer Diameter


Inner Diameter


Workflow
inactive
user
active
user input
not accepted
problem definition
accepted
feasibility
yes
use sensitivity?
no
sensitivity analysis
optimization
optimization
satisfied
post-processing
not satisfied

Classes: Modules

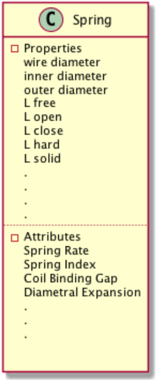


Figure : Spring Class

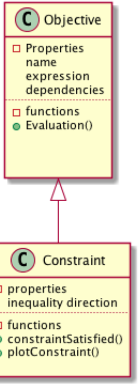
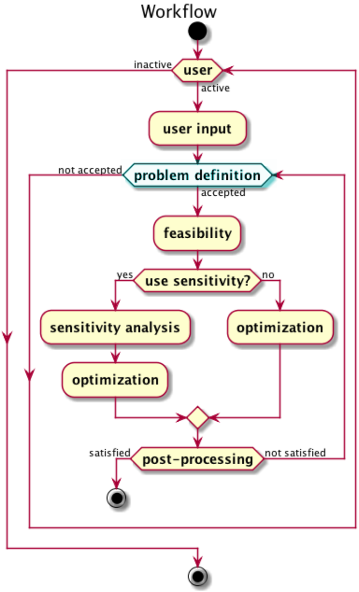


Figure : Objective and Constraint Classes

Mathematical Formulation

$$\min_{\mathbf{x}} \quad F(\mathbf{x}),$$
$$\text{subject to} \quad \mathbf{G}(\mathbf{x}) \leq 0$$

- $F(\mathbf{X})$: The objective function
- $\mathbf{G}(\mathbf{x})$: The set of constraints.



Feasibility

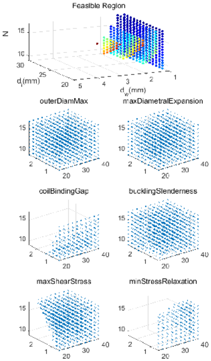
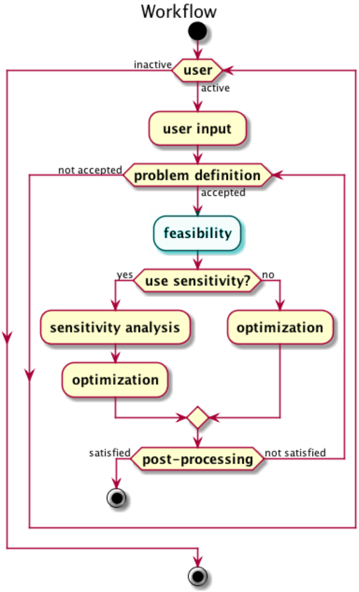
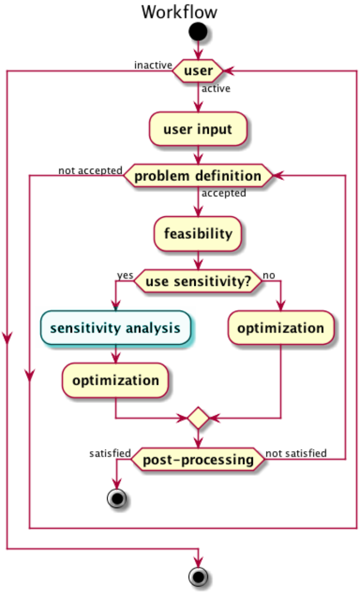


Figure : Feasibility Region



Sensitivity Analysis

- If the dimension of the design space increases then computation expense increases.
- Dimension Reduction
- Measure of influence
- Locally $\rightarrow$ around a point (Gradient).
- Globally $\rightarrow$ Overall sensitivity (Sobol Indices).
- $S_i = \frac{\mathbf{Var}_i(\mathbf{E}_{-i}(Y|X_i))}{\mathbf{Var}(Y)}$

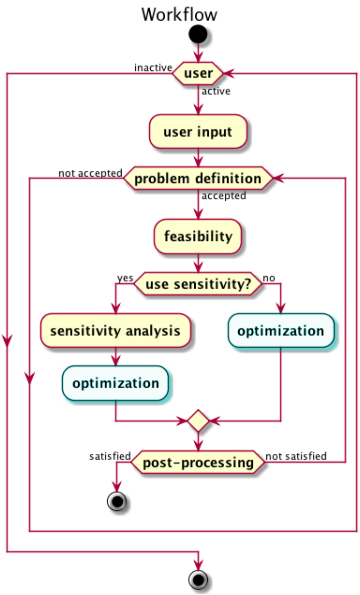


Optimization

- Constrained Optimization

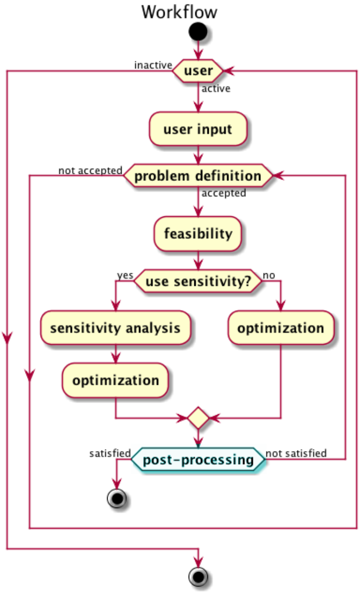
$$\min_{\mathbf{x}} \quad F(\mathbf{x}),$$
$$\text{subject to} \quad \mathbf{G}(\mathbf{x}) \leq 0$$

- DIRECT algorithm:
  sampling-based, derivative-free
- Easily integrated with the workflow.



Post-Processing

- Prompt user with results of optimization.
- User can accept, reject the results and redefine problem definition.



Stress Relaxation and Creep

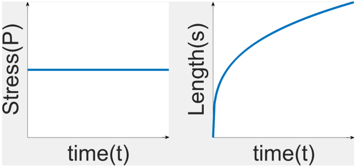


Figure : Creep

$$s(t) - s(0) = \left(\frac{(d_\mathrm{i}+d_\mathrm{w})\varepsilon}{\pi} \frac{4+3n}{n+1} \right)^{n+1} \frac{2^n \pi (d_\mathrm{i}+d_\mathrm{w})^2 N_\mathrm{a} c}{k d_\mathrm{w}^{4+3n}} t^k$$

- Constant stress
- Deflection increases with time

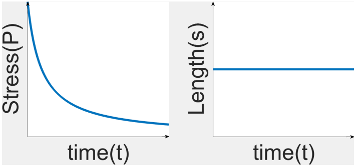


Figure : Stress Relaxation

$$\Phi = \frac{2\pi N_\mathrm{a} (d_\mathrm{i} + d_\mathrm{w})^2}{G s d_\mathrm{w}^4} A$$

$$A = \int_0^{d_\mathrm{w}} r^2 \left(\left(\frac{2Gsr}{\pi N_\mathrm{a} (d_\mathrm{i} + d_\mathrm{w})^2} \right)^{-n} + \frac{c}{k} G n t^k \right)^{-\frac{1}{n}} \mathrm{d}r$$

where c, k, n are temperature and material dependent constants.

- Constant strain
- Stress decreases with time

Design Optimization Under Uncertainty

- Variability in manufacturing process $\rightarrow$ tolerance/uncertainty.
- Considerations in design process:

$$\min_{x} \ \mathbf{E}(F(\mathbf{x}, d)),$$

such that

$$Prob(G(\mathbf{x}, d) < 0) > \rho_t$$

$$Prob(\mathbf{x} > lb_{\mathbf{x}}) > \rho_t^{lb}$$

$$Prob(\mathbf{x} < \mu b_{\mathbf{x}}) > \rho_t^{ub}$$

- Each iteration $\rightarrow$ Probabilistic constraints checked using Monte Carlo.
- UQ propagation with Monte Carlo to obtain $F(\mathbf{x}, d)$

- Contributions:
  - Flexible algorithm for spring design optimization, with a variety of objective functions and constraints.
  - Incorporated models for stress relaxation and creep into optimization.
  - Performed spring design optimization under uncertainty
- Future Work
  - Analysis of different stress relaxation and creep models.
  - Practical testing and validation of the interface.

Thank you.

Questions?

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