TEACHING VS. LEARNING IN MATHEMATICS

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Contents

Introduction 1
1. Goals vs. Responsibilities 1
1.1. Generalities 1
1.2. Calculator arithmetic 2
1.3. Computer calculus? 3
2. Improved teaching vs. Improved learning 4
3. Computer teaching vs. Computer–based learning 4
4. Information delivery vs. Diagnosis 5
5. Summary 5

Introduction

Most educators see teaching and learning as two sides of the same coin: we teach so they will learn, end of story. It was hard to compare while everyone was doing more-or-less the same thing. Technology has changed this however, and I'll give examples that suggest we are far too focused on what happens on our side of the desk. It looks as though teaching and learning were never as closely linked as we wanted to think, and the gap will widen unless we really focus on students and learning, particularly long–term learning, and *not* through the lens of teaching[1].

1. Goals vs. Responsibilities

The way we organize it, math begins with arithmetic and the rest of the subject is built on this. Arithmetic instruction should, therefore, provide a foundation for learning in the rest of mathematics. We need some careful terminology to describe how this should work.

1.1. **Generalities.** Teaching or learning *goals*are usually understood as short–term, specifying deliverables, and determined by the teacher or course designer. Teaching goals specify that a teacher should do certain things, while learning goals specify that students should end up with certain things. Traditional goals in arithmetic mostly concern working problems.

The objective "provide a foundation for further learning" does not qualify as a goal in this sense so we refer to it as a *responsibility.* More explicitly, responsibilities are long–term or downstream, defined operationally rather than explicitly, and not

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[1]This is not a new point, see Association for Educational Communications and Technology. We, as a community, might have avoided a lot of grief if we had paid more attention to it.

a matter of choice. In principle, goals should be chosen so that responsibilities are fulfilled. It is certainly not clear how "work problem" goals end up meeting "provide foundation" responsibilities, but in traditional courses it seems to work.

A final, and absolutely vital, general point is that "students" are individuals. We have goals for and responsibilities to individual students, and different individuals might need substantially different goals and responsibilities. Discussions that don't stay grounded in this reality encourage one-size-fits-all thinking that is a real disservice to students.

1.2. **Calculator arithmetic.** Returning to arithmetic, there have been two recent developments. First, calculators enable more students to achieve "work problems" goals more easily and with greater accuracy; and second, "understand what they are doing" has replaced rote computation as a goal[2]. The good news is that in these programs goals are being met better than ever. The bad news is that long–term responsibilities are not being met. Number-sense and symbolic-skills deficits in students from these programs were a major issue in the K–12 "math wars" and are a serious concern at the college level.

Apparently a disconnect developed between goals and responsibilities. What happened and what can we learn from it?

The first lesson is that "work problems" by itself is evidently not enough to "provide a foundation". Apparently there was something about the *way* traditional students work problems that was important[3]. But rejecting calculators is not a satisfactory response. We urgently need to understand *how* by–hand arithmetic supports later learning. Perhaps we can fix the calculator approach by adding the missing factor to teaching goals. This might improve the traditional approach as well[4].

The second lesson is that there are several ways to address responsibility problems. Responsibilities of one level can be thought of as preparing students to accomplish goals at the next level. If goal changes at the lower level no longer meet this responsibility then one response is to adjust the lower–level goals. However it is also possible to change the definition of "responsibility" by *changing the goals* of the higher level. This was the strategy in K–12 calculator-oriented curricula. They adjusted goals at all levels to "take advantage of calculator skills" and de–emphasize traditional goals not supported by calculators. The result was a system with internally consistent goals and responsibilities.

The goal–changing approach to responsibility eventually fails. College courses have responsibility for preparation for study in science, engineering and advanced mathematics. These responsibilities are determined by the demands of the subjects and can't be negotiated. Meeting these responsibilities strongly constrains choices of short-term goals in college courses. Working down the chain, college course goals should establish end-of-curriculum responsibilities for K–12. There we have a train wreck: the calculator–oriented K–12 community (at least) seems to have no understanding of, nor interest in, these external responsibilities.

So far this lesson seems to concern responsibility, but there is a teaching/learning core. *Responsibilities* concern learning because the teacher is not in the picture

[2]Reference to NCTM standards?

[3]For a guess see "K–12 calculator woes".

[4]If the guess in the previous footnote is right then traditional approaches are indeed far from optimal.

when responsibilities fall due. However the K–12 education community is intensely teacher–oriented. The “responsibility” idea is not part of the world view and even hard to formulate sensibly.

A comforting corollary of this last point is that the school/college mismatch result from a lack of understanding rather than conscious irresponsibility. This is further illustrated by a common K-12 response to college–level complaints: we should follow their lead and adjust our teaching goals to “take advantage of new skills” rather than bemoan the decline of old ones. Our unwillingness to do so looks like a reactionary attachment to the past; it doesn’t occur to them that it might result from constraints of downstream responsibilities.

The third lesson from arithmetic concerns why taking understanding as a teaching goal did not improve outcomes, and in particular why it did not replace the mysterious benefit of hand arithmetic. The reason is not deep. Over the millennia mathematicians have found that in order to support learning “understand” must be given a rather strong meaning, including “make the solving of problems straightforward”. K–12 educators use the word in a much weak sense that does not imply skills. They use a meaning already known (by mathematicians) to be dysfunctional for mathematical learning!

To connect this to the teaching/learning theme note that the teaching point of view suggests a lot of flexibility in choosing goals. One can choose the meaning for “understand” and there is no obvious reason why one should not take a weak one that is easy to achieve. Furthermore one can formulate teaching goals to address any definition.

The learning point of view is much more constrained. First, the meaning used for a word must accomplish longer–term responsibilities. Second, in order to incorporate something into learning goals it must be visible in outcomes, i.e. be testable in some way. Even *trying* to use the math-ed definition for “understand” would have revealed it’s inadequacy. This doesn’t solve the problem though. The mathematical meaning satisfies responsibilities and is testable but is far too demanding to be useful at this level. We are taken back to the point in the first lesson: we need a functional explanation of *how* by–hand arithmetic supports later learning before we can identify good learning goals.

1.3. **Computer calculus?** I used calculator arithmetic in this example because the response of the K–12 community amplified rather than fixed the problem and consequently it gives a relatively clear signal at the college level. Similar things can happen at any level. For instance, do calculus students learn more from by-hand techniques of integration than just how to evaluate integrals? Use of computer integration packages will let us meet teaching goals more easily and more often, but will they undermine long–term learning?

I have learned that I am not smart enough to anticipate something like the calculator–arithmetic problem. Without this example I might have used computers to screw up my integral calculus course and not known there was a responsibility failure unless someone at the graduate level or in a client discipline pointed it out. But I should be smart enough to learn from the example. If I screw up integral calculus now it will be my fault.

2. Improved teaching vs. Improved learning

College classes over 100 are now common. Traditional teacher–student interactions are impossible, but a number of technologies have been developed to provide substitutes. One is classroom polling (clickers). A recent article in the journal Science[5] describes how a biology professor used clickers to show that students can learn by talking to each other. Another approach assumes students have laptop computers, as is now common in our engineering courses. Software enables the professor to send material to all students or specific students; receive questions or comments from students; import material from student computers to assess later or to display and discuss, etc. Some professors are quite enthusiastic about this.

These technologies improve the teaching experience. Do they improve learning?

The first point is that 100 students were crammed in the room for economic reasons: the professor's time is so valuable that we cannot afford to split the class into two sections of 50, or three sections of 34. We know that students learn less in big classes so we try to restrict the practice to courses with modest goals. Nonetheless there is a question: is getting students to talk to each other really the best use of precious class time[6]?

The second point is that when a teacher interacts with one student he is to some degree neglecting the others. Interacting with one in a class of 100 is to neglect 99. It may not be quite this bad: if ten students are interested in the interaction, and only 50 were engaged anyway, then student engagement decreases only by a factor of 5. But it might be worse. I've now spent years in one-on-one help sessions, and looking back I suspect I missed the point of half the questions I was asked in class, and I fear that 20% of my responses probably disengaged *everyone*.

In other words teacher–student interactions in large classes are almost always a massive waste of student time. It may improve teaching in a superficial sense but does not contribute to learning.

The extreme learning–oriented view is this: think of the teacher's time, or maybe the teacher's salary, as a resource. Is a traditional class the best way to use this resource to get learning? In some cases there are already computer-based systems that would do better. The message I think we should be getting, in math anyway, is that there is a point beyond which teaching in the traditional sense is no longer a satisfactory path to learning.

3. Computer teaching vs. Computer–based learning

Most courseware is developed by experienced educators, which is to say people with a lot of classroom expertise. It shows: most computer courses are modeled on traditional courses and the computer is seen as an "electronic teacher".

Ten years spent watching students trying to deal with courseware has convinced me that this point of view is wrong. Students have to take an active role in computer-based learning. They seem to have "learning instincts" in the sense that there are consistent behaviors when they are ready to go to the next stage, get stuck, etc. Sometimes there are several different patterns. The point is that none of these patterns match classroom practice.

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[5] "Why peer discussion improves student performance on in-class concept questions", Science, vol. 323 pp. 122–124.

[6] The benefits of student interactions are not in question and did not need rediscovery. But shouldn't we try to promote it outside the classroom?

We have to think of the learner as the center of the process. Not think "what should we have her do next" but "how might she want to approach the next task?" Watch and find out rather than extrapolate from classroom experience. And then make sure the way is clear and tools designed to work the way she wants to use them are at hand.

4. Information delivery vs. Diagnosis

What is a teacher's core mission? Most would give some version of "information delivery" and most classroom practices fit this description.

Students now have many sources of information. I have seen students look something up on Wikipedia rather than try to find it in the course text. Web materials and computer courseware can do a good job of providing information in a variety of media and at convenient times. Are teachers irrelevant, or is there a better description of the mission?

I believe our principal mission should be "help with *problems* of information delivery". Students learn relatively easily but the learning is usually flawed. What we can do that machines cannot is diagnose and fix learning errors. The key, again, is a shift of emphasis from teacher to learner.

The computer–side help system in the Math Emporium[7] illustrates this point. In a nutshell the help goal is "fix and run". The helper listens carefully to diagnose the student's specific problem, says the minimum needed to get them past it, and leaves.

Experienced teachers have a hard time doing fix-and-run. They want to say "let me explain this to you" and give a mini-lecture. The answer to the student's problem is in there somewhere but neither the teacher nor the student know where. The teacher didn't diagnose the specific problem, and the student has probably already heard a lecture that didn't work. Or the teacher will say "I'll show you how to work this problem". The student's work, good as well as bad, is discarded. The new solution may help but the student is often left with a flaw that will surface again later. It is very hard for experienced teachers to listen instead of talk, but this is the key to learner–oriented education.

I myself have thirty years of classroom teaching whispering in my ear "give your insightful lecture". As with advice to my children about their boyfriends and girlfriends, I've had to learn that an insightful lecture is often not the best path to learning.

5. Summary

Technology has enabled us to make some pretty bad mistakes. In the long run this is all right if we recognize and correct these mistakes. But one of the lessons seems to go to the very core of the way we see ourselves: teaching is not the same as learning, and changes that we think improve teaching may actually degrade learning. Can we make the transition from "teachers" to "learning facilitators"?

[7] Virginia Tech Math Emporium, http://www.emporium.vt.edu