Turbulent Supersonic Jet Noise

a life-size paradigm for model reduction of transport-dominated phenomena

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never without
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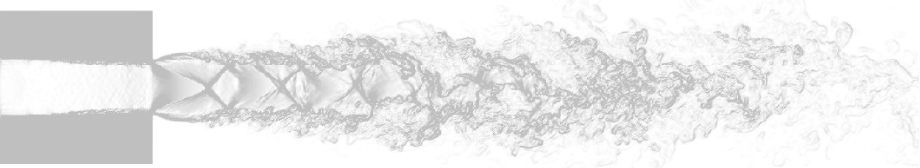
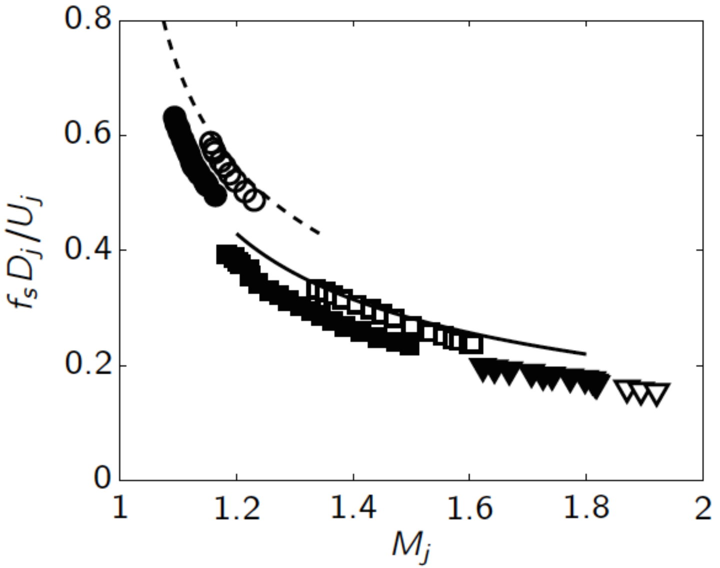


Motivation



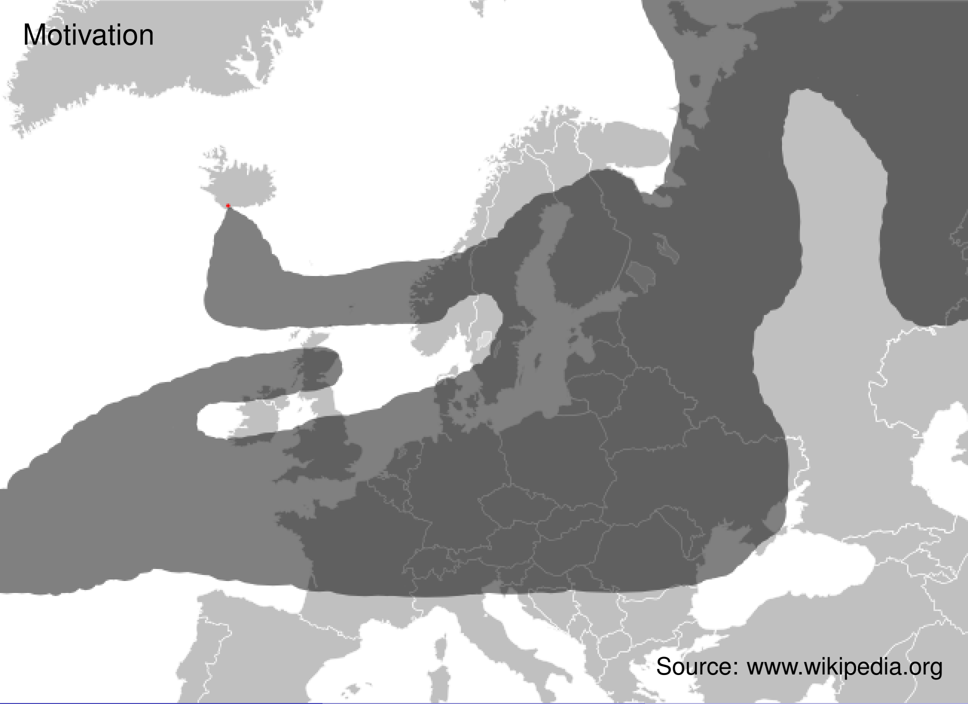
Motivation

Source: www.nasa.gov

Motivation

Source: www.jongustafsson.com

Motivation



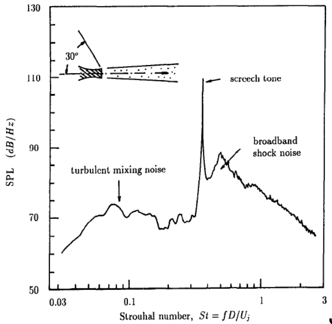
Source: www.wikipedia.org

Stromboli 2013

Stromboli 2013, Photo: Daniele Andronico INGV Catania

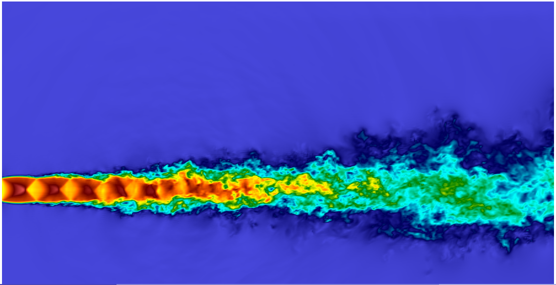
Acoustics of a Free Jet

- turbulent mixing noise
- broadband shock noise
- screech

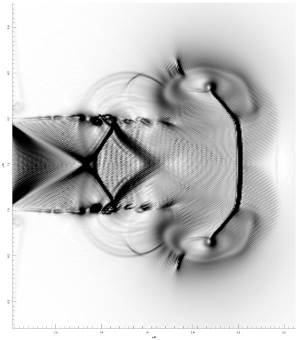


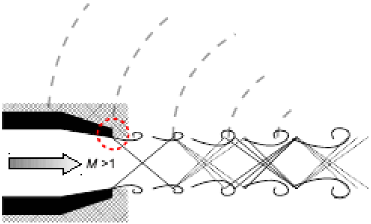
Jet

Noise Spectrum (Tam 1995)

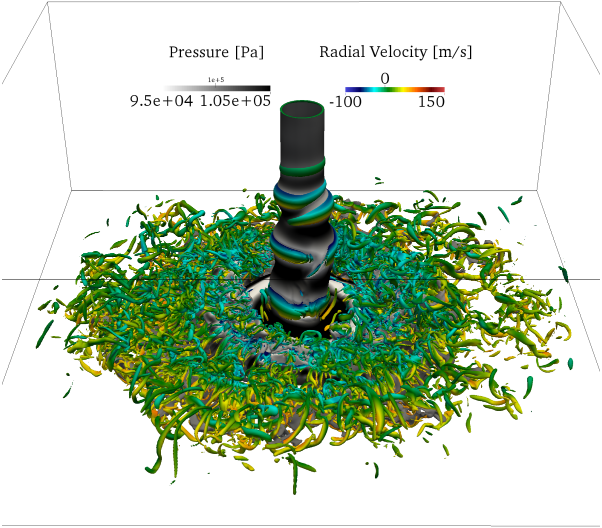


Acoustics of a Free Jet
Shock Shear Layer Interaction



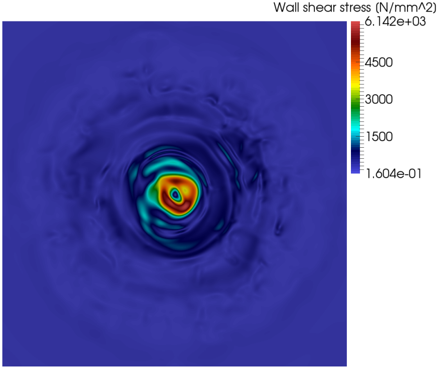


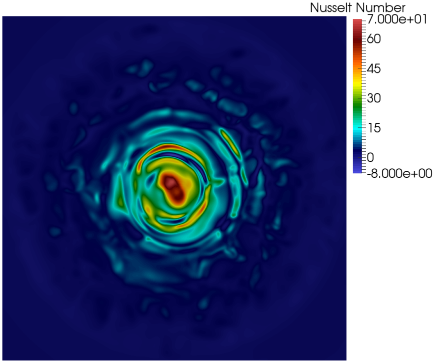
The Impinging Jet



Iso-surfaces at $Ma = 0.2$: pressure and at $Q = 10^5 m^2 s^{-4}$

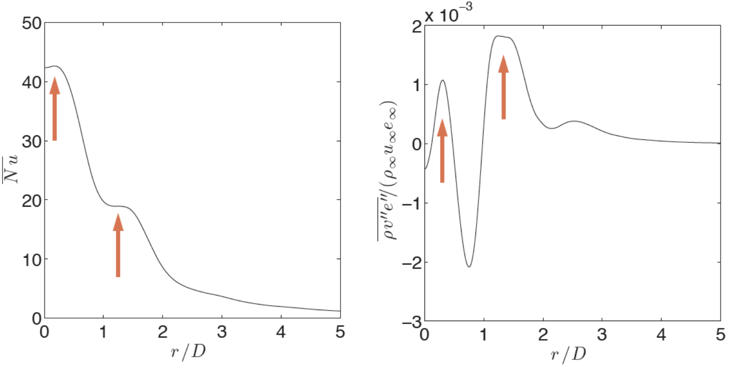
The Impinging Jet
Wall shear stress and Nusselt number





The Impinging Jet

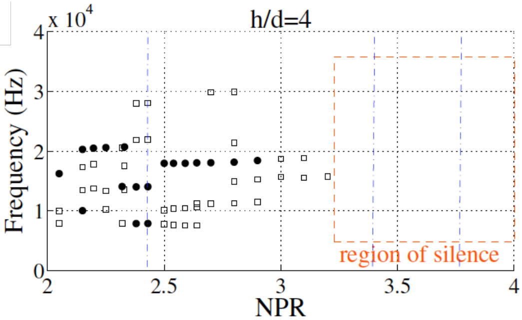
Nusselt number and turbulent heat flux



Acoustics of a supersonic impinging jet
Impinging tones

Possible mechanisms

- Screech for $h/D \geq 5$
- Standoff Shock oscillation
- Shear Layer Instability

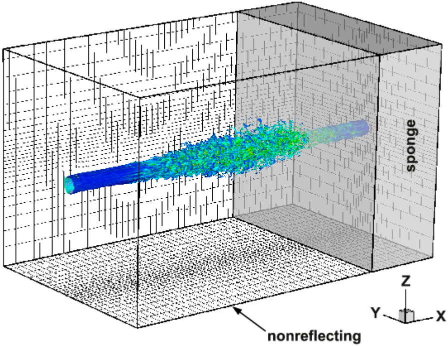


Impinging Tones
Sinibaldi et al., 2014

Details of the Numerical Simulation

- compressible Navier–Stokes equations
- 6. order in space (compact)
- 4. order Runge-Kutta / exponential Krylov in time
- Domain of free jet: $24D \times 12D \times 12D$
- Domain of impinging jet: $5D \times 12D \times 12D$

- Grid of free jet: $2048 \times 1024 \times 1024$
- Grid of impinging jet: $1024 \times 1024 \times 1024$



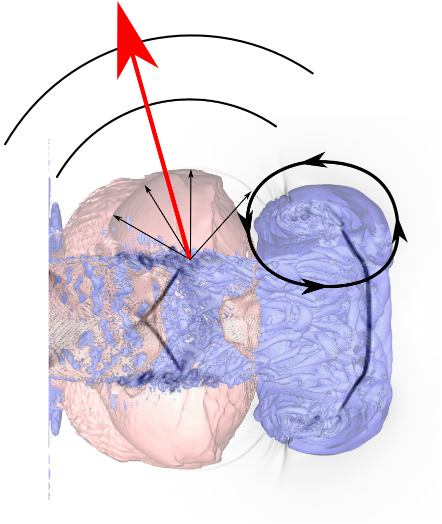
Goal of the talk

- Jets are tremensously expensive to compute
- the main players are
  - jet modes
  - shocks
- Targets of interest
  - Sound pressure level
  - Particle Load
  - Mass Flow
  - Heat Transfer
- Target seems to depend on those “simple” structures
  - model dependency of targets by Ma, Re
  - identify parameters by simple measurements

Acoustics of a Starting Jet

Trailing jet

Vortex ring

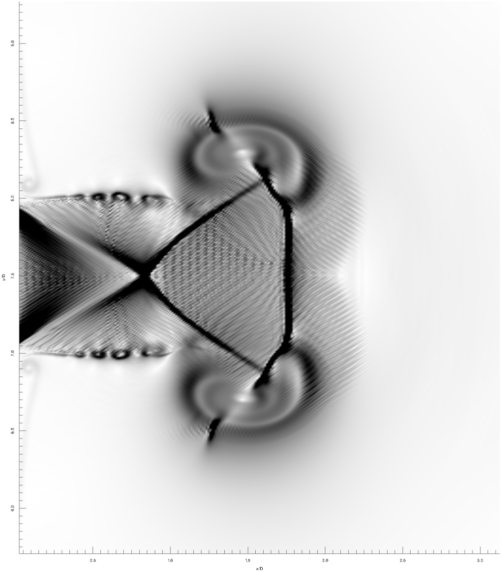


Shock-waves

Radiated noise

Acoustics of a Starting Jet

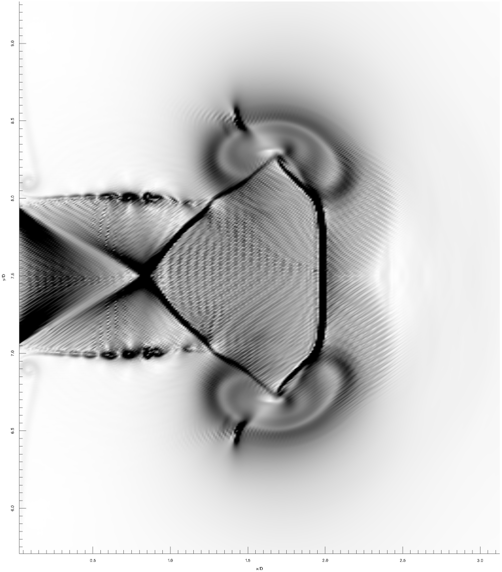
Interaction shear layer - shock-wave - vortex ring



straight shock

Acoustics of a Starting Jet

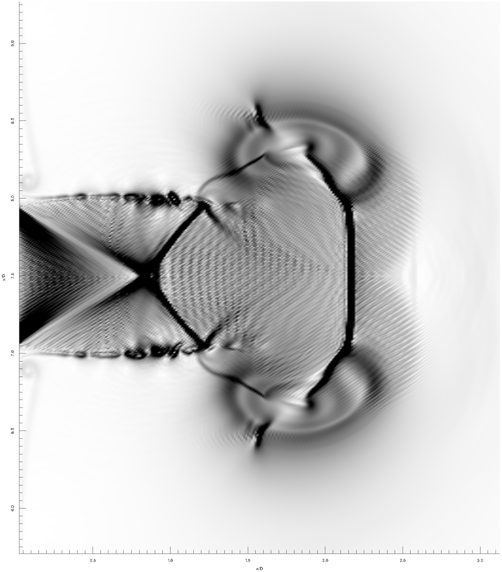
Interaction shear layer - shock-wave - vortex ring



curved shock

Acoustics of a Starting Jet

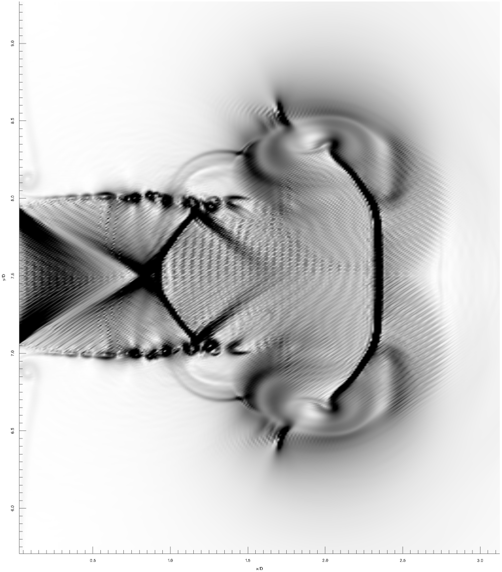
Interaction shear layer - shock-wave - vortex ring



curved and reflected shock

Acoustics of a Starting Jet

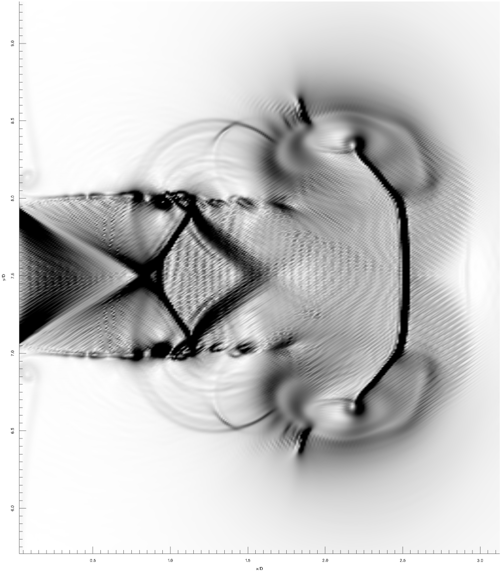
Interaction shear layer - shock-wave - vortex ring



radiated shock

Acoustics of a Starting Jet

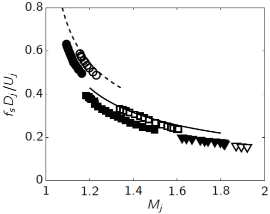
Interaction shear layer - shock-wave - vortex ring



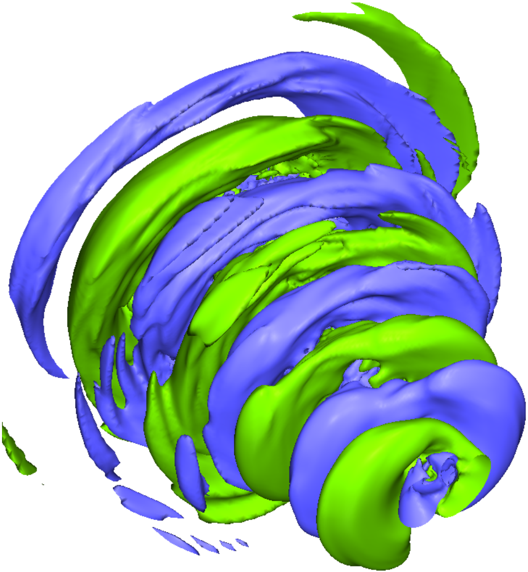
open shock

Acoustics of a Free Jet

Modal Structure

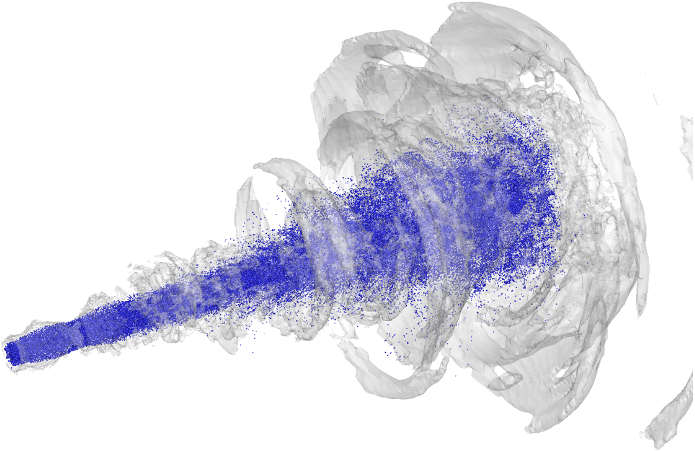


Principal jet modes



Acoustics of a Free Jet

associated Sound Radiation



Particle flow

Jet noise modification due to Particles

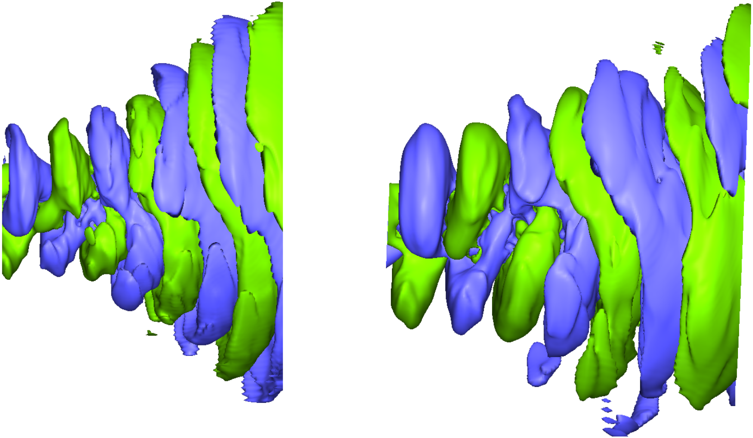


Figure: Vortical Modes, without and with particles

Particle flow

Jet noise modification due to Particles

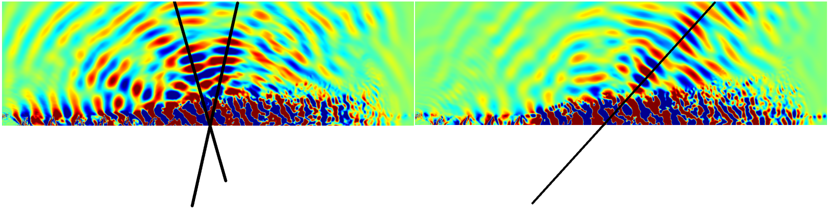


Figure: Vortical Modes, without and with particles

Particle flow

Jet noise modification due to Particles

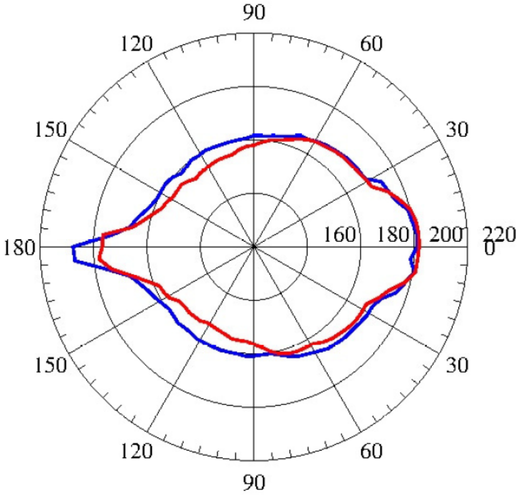
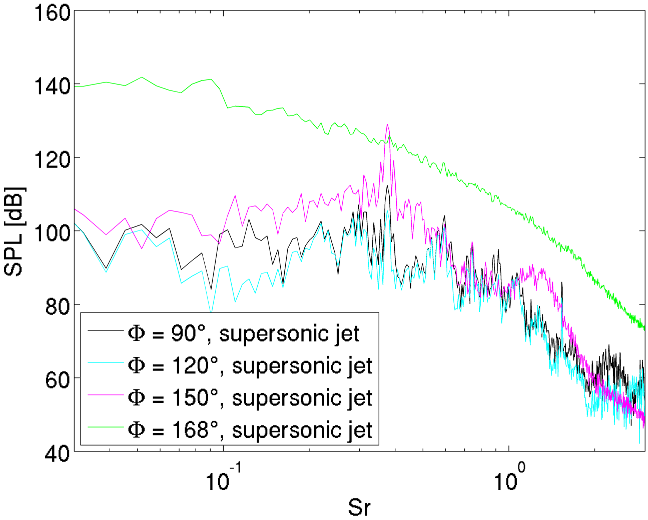


Figure: Vortical Modes, without and with particles

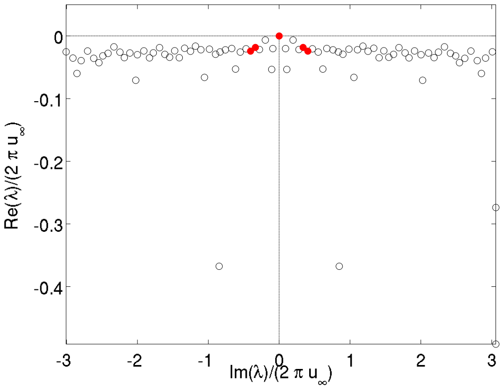
Acoustics of a Free Jet

Sound pressure level at different observation angles



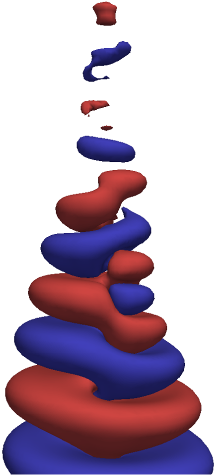
Modal Decomposition of a supersonic Free Jet

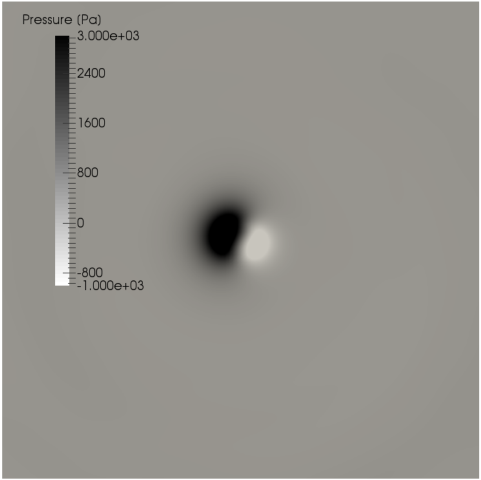
Distribution of Eigenvalues



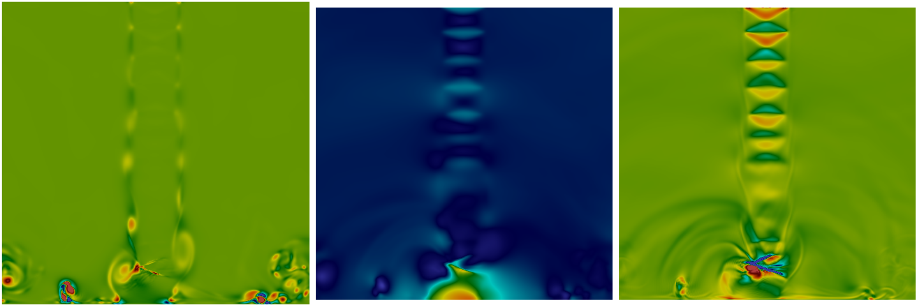
Modal Decomposition of a supersonic Free Jet

Dynamic Mode at $Sr = 0.37$





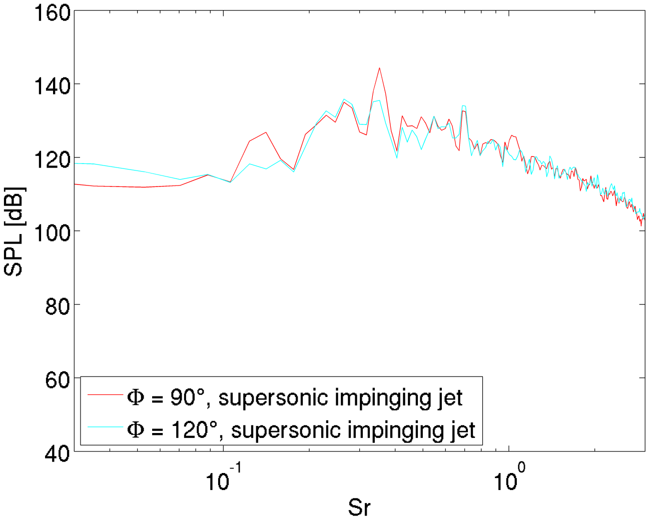
Flow field of a supersonic Impinging Jet



left: Q middle: p right: $div(u)$

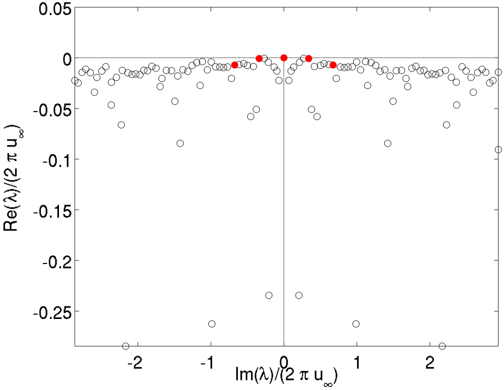
Acoustics of a supersonic impinging jet

Sound pressure level at different observation angles



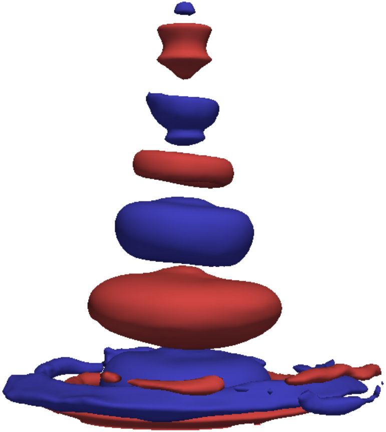
Modal decomposition of a supersonic impinging jet

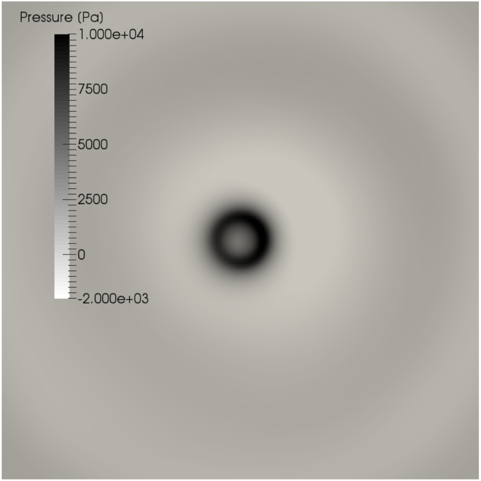
Distribution of Eigenvalues



Modal decomposition of a supersonic impinging jet

Dynamic Mode at $Sr = 0.35$ (0.70)





Objective

Basic concept

Experiment $\rightarrow$ Data assimilation $\leftarrow$ Model

$\uparrow$

Difference between computed and measured states is to be minimized

Objective

Basic concept

Experiment → Data assimilation ← Model

↑

$$\min\left(J\right) = \int_{t_0}^{t_1} \iint_{S_p} \left(u_i - u_{i,\mathrm{PIV}}\right)^2 \,\mathrm{d}S_p\,\mathrm{d}t$$

Objective function

$$J = \frac{1}{2} \int_{\Omega} \int_{t_0}^{t_{\text{end}}} (q - q_{\text{tar}})^2 \, d\Omega$$

$$\delta J = \int_{\Omega} \int \underbrace{(q - q_{\text{tar}})}_{g} \delta q d\Omega$$

Constrained optimisation

Linearised Navier-Stokes equations

$$N \delta q = \delta f$$

Adjoint equations
Introductory example

Objective function

$$J = \frac{1}{2}\int_{\Omega}\int_{t_0}^{t_{\mathrm{end}}} (q - q_{\mathrm{tar}})^2 \, \mathrm{d}\Omega$$

$$\delta J = \int_{\Omega}\int \underbrace{(q - q_{\mathrm{tar}})}_{g} \delta q \mathrm{d}\Omega$$

Constrained optimisation

Linearised Navier-Stokes equations

$$N\delta q = \delta f$$

Adjoint equations

Introductory example

Lagrangian approach

$$\begin{aligned} \delta J &= g^T \delta q - (q^*)^T \underbrace{(N\delta q - \delta f)}_{=0} \\ &= \delta q^T \underbrace{(g - N^T q^*)}_{=0} + (q^*)^T \delta f \end{aligned}$$

Adjoint equations
Introductory example

Lagrangian approach

$$\delta J = g^T \delta q - (q^*)^T \underbrace{(N\delta q - \delta f)}_{=0}$$
$$= \delta q^T \underbrace{(g - N^T q^*)}_{=0} + (q^*)^T \delta f$$

Change of J becomes simply

$$\delta J = {q^*}^T \delta f \rightarrow \frac{\delta J}{\delta f} = q^* \approx \nabla_f J$$

Adjoint equations

Introductory example

Lagrangian approach

$$
\begin{aligned}
\delta J &= g^T \delta q - (q^*)^T \underbrace{(N\delta q - \delta f)}_{=0} \\
&= \delta q^T \underbrace{(g - N^T q^*)}_{=0} + (q^*)^T \delta f
\end{aligned}
$$

Change of J becomes simply

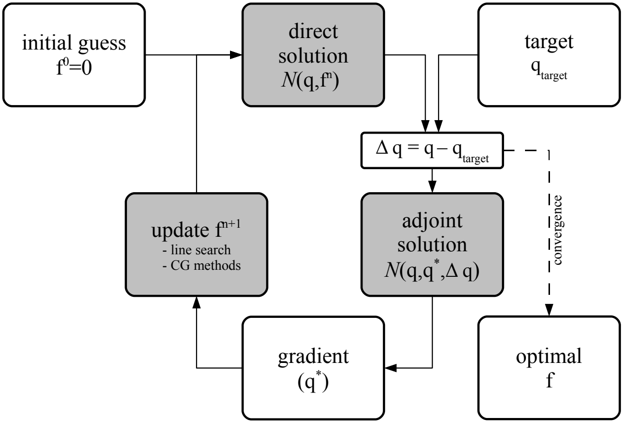
$$
\delta J = {q^*}^T \delta f \rightarrow \frac{\delta J}{\delta f} = q^* \approx \nabla_f J
$$

Adjoint equations

Iterative procedure

Optimal change of f

$$f^{n+1} = f^n + \nabla_f J$$



Euler equations

$$\partial_t \begin{pmatrix} \varrho \\ \varrho u_j \\ \frac{p}{\gamma-1} \end{pmatrix} + \partial_{x_i} \begin{pmatrix} \varrho u_i \\ \varrho u_i u_j + p\delta_{ij} \\ \frac{u_i p \gamma}{\gamma-1} \end{pmatrix} - u_i \partial_{x_i} \begin{pmatrix} 0 \\ 0 \\ p \end{pmatrix} = f$$

$$\partial_t a + \partial_{x_i} b^i + C^i \partial_{x_i} c = f.$$

Adjoint equations
Compressible Navier-Stokes

Linerisation

$$\partial_t \underbrace{\frac{\partial a_\alpha}{\partial q_\beta}}_{A} \delta q_\beta + \partial_{x_i} \underbrace{\frac{\partial b_\alpha}{\partial q_\beta}}_{B^i} \delta q_\beta + C^i \partial_{x_i} \delta q_\beta + \delta C^i \partial_{x_i} c_\beta = \delta f$$

$q = [\varrho, u_j, p]$

Lagrangian approach

$$\iint \delta J \mathrm{d}\Omega = \iint g^T \delta q \mathrm{d}\Omega$$
$$- \iint {q^*}^T \underbrace{\left(\partial_t A \delta q + \partial_{x_i} B^i \delta q + C^i \partial_{x_i} \delta q + \delta C^i \partial_{x_i} c - \delta f\right)}_{=0} \mathrm{d}\Omega$$

$\mathrm{d}\Omega = \mathrm{d}x_i \mathrm{d}t$ is the space-time measure

Integration by parts

$$
\begin{aligned}
\iint \delta J \mathrm{d}\Omega = & \iint \delta q^T g \mathrm{d}\Omega \\
& + \iint \delta q^T A^T \partial_t q^* \mathrm{d}\Omega && - \left[\int \delta q^T A^T q^* \mathrm{d}x_i \right]_{t=t_0}^{t=t_{\mathrm{end}}} \\
& + \iint \delta q^T {B^i}^T \partial_{x_i} q^* \mathrm{d}\Omega && - \left[\int \delta q^T {B^i}^T q^* \mathrm{d}t \right]_{x_i=x_{i,0}}^{x_i=L_i} \\
& + \iint \delta q^T \partial_{x_i} {C^i}^T q^* \mathrm{d}\Omega && - \left[\int \delta q^T {C^i}^T q^* \mathrm{d}t \right]_{x_i=x_{i,0}}^{x_i=L_i} \\
& - \iint \delta q^T \tilde{C}^i \partial_{x_i} c && q^*_\alpha \delta C^i_{\alpha\beta} \partial_{x_i} c_\beta = q^*_\alpha \delta q_\kappa \frac{\partial C^i_{\alpha\beta}}{\partial q_\kappa} \partial_{x_i} c_\beta \\
& + \iint {q^*}^T \delta f \mathrm{d}\Omega && \delta q_\kappa \tilde{C}^i_{\kappa\beta} \partial_{x_i} c_\beta
\end{aligned}
$$

Adjoint equations

Compressible Navier-Stokes

Factor out different variations

$$
\begin{aligned}
\iint \delta J \mathrm{d}\Omega = & \iint {q^*}^T \delta f \mathrm{d}\Omega \\
& + \iint \delta q^T \underbrace{\left(g + A^T \partial_t q^* + {B^i}^T \partial_{x_i} q^* + \partial_{x_i} {C^i}^T q^* - \tilde{C}^i \partial_{x_i} c \right)}_{I} \mathrm{d}\Omega \\
& \underbrace{- \left[\int \delta q^T A^T q^* \mathrm{d}x_i \right]_{t=t_0}^{t=t_{\text{end}}}}_{II} \\
& \underbrace{- \left[\int \delta q^T {B^i}^T q^* \mathrm{d}t \right]_{x_i=x_{i,0}}^{x_i=L_i} - \left[\int \delta q^T {C^i}^T q^* \mathrm{d}t \right]_{x_i=x_{i,0}}^{x_i=L_i}}_{III}
\end{aligned}
$$

Term I: Resulting adjoint equations

$$\partial_t q^* = -{A^T}^{-1} {B^i}^T \partial_{x_i} q^* - {A^T}^{-1} \partial_{x_i} {C^i}^T q^* + {A^T}^{-1} \tilde{C}^i \partial_{x_i} c - {A^T}^{-1} g$$

Navier-Stokes:

$$\begin{aligned} &- A^T \partial_t q^* - {B^i}^T \partial_{x_i} q^* - \partial_{x_i} {C^i}^T q^* + \tilde{C}^i \partial_{x_i} c = \\ &+ \partial_{x_j} D^T \partial_{x_i} q^* - {E^i}^T \partial_{x_i} q^* + {F^{II}}^T \partial_{x_i} {F^I}^T \partial_{x_i} q^* - \partial_{x_i} {G^i}^T q^* + H^T q^* + g \end{aligned}$$

Adjoint equations

Lagrangian approach

$$\delta J = g^T \delta q - {q^*}^T \left(\partial_t A \delta q + \partial_x B \delta q - \delta f\right)$$
$$- {l^*}^T \left(\delta q(0,t) - \delta l(t)\right) \quad \text{(left boundary)}$$
$$- {r^*}^T \left(\delta q(L,t) - \delta r(t)\right) \quad \text{(right boundary)}$$

Inclusion of boundary conditions (1D simplification)

$$q(x=0,t) = l(t)$$
$$q(x=L,t) = r(t)$$

Adjoint equations

$$\delta J = \delta q^T \underbrace{\left(g + A^T \partial_t q^* + B^T \partial_x q^*\right)}_{=0} + {q^*}^T \delta f$$

$$- \left[\delta q^T A^T q^*\right]_{t=0}^{t=T} \qquad \text{(adjoint initial condition)}$$

$$- \left[\delta q^T B^T q^*\right]_{x=0}^{x=L} \qquad \text{(adjoint boundary condition)}$$

$$- \delta q(0,t)^T l^* + {l^*}^T \delta l(t)$$

$$- \delta q(L,t)^T r^* + {r^*}^T \delta r(t)$$

Simplified derivation

$$\delta J = \delta q^T \underbrace{\left(g + A^T \partial_t q^* + B^T \partial_x q^*\right)}_{=0} + {q^*}^T \delta f$$

$$+ \delta q(0,t)^T \underbrace{\left(+B^T q^*(0,t) - l^*\right)}_{=0} + {l^*}^T \delta l(t)$$

$$+ \delta q(L,t)^T \underbrace{\left(-B^T q^*(L,t) - r^*\right)}_{=0} + {r^*}^T \delta r(t)$$

Resulting sensitivities

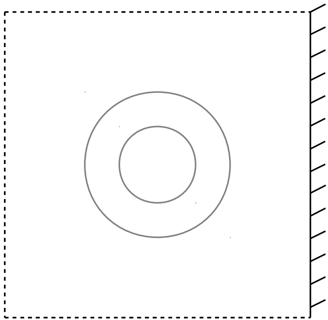
$$\frac{\delta J}{\delta l} = l^* = -B^T q^*(0,t) \approx \nabla_l J$$
$$\frac{\delta J}{\delta r} = r^* = +B^T q^*(L,t) \approx \nabla_r J$$

Optimal change of the boundaries is defined by the adjoint solution

Adjoint based data assimilation of NSCBC

Example

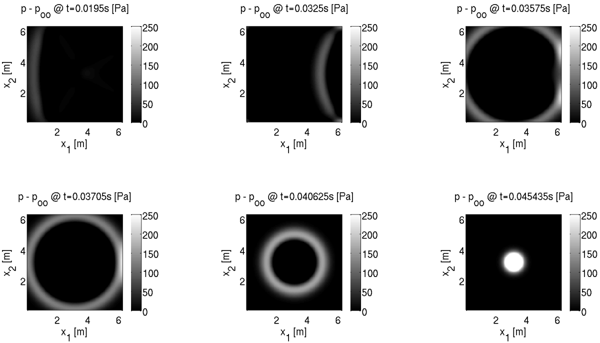
Setup



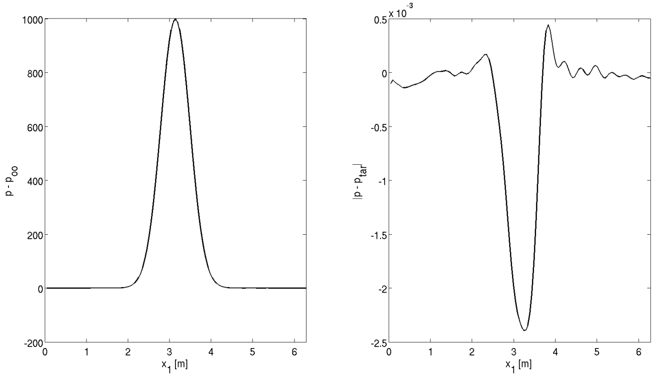
- Euler equations (2D)
- Non-reflecting bounds
- Slip wall on the right

- Target: formation of a pressure pulse @ t=T
- Control: Non-reflecting bounds

Results

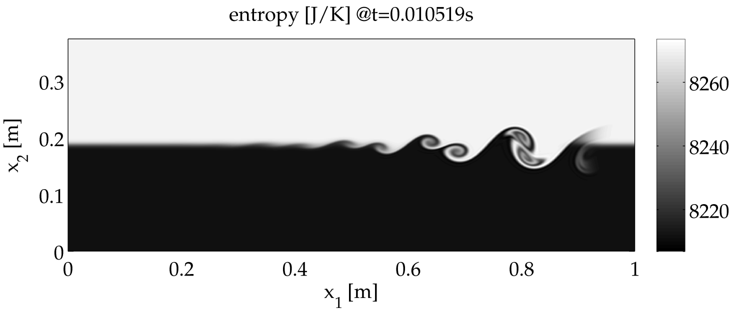

p - p$_{oo}$ @ t=0.0195s [Pa]
p - p$_{oo}$ @ t=0.0325s [Pa]
p - p$_{oo}$ @ t=0.03575s [Pa]
p - p$_{oo}$ @ t=0.03705s [Pa]
p - p$_{oo}$ @ t=0.040625s [Pa]
p - p$_{oo}$ @ t=0.045435s [Pa]
x$_1$ [m]
x$_2$ [m]
2
4
6
0
50
100
150
200
250

Results

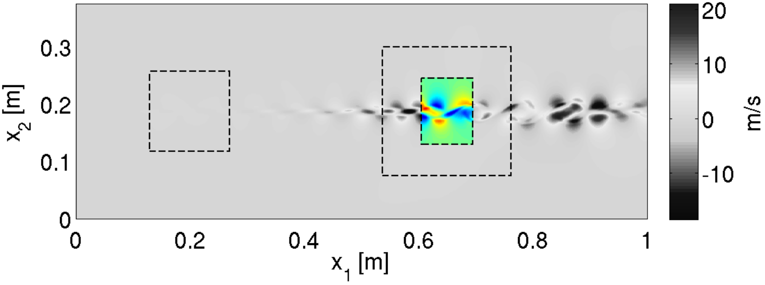


1000
800
600
400
200
0
-200
0 1 2 3 4 5 6
x_1 [m]
p - p_{oo}
x 10^{-3}
0.5
0
-0.5
-1
-1.5
-2
-2.5
0 1 2 3 4 5 6
x_1 [m]
|p - p_{tar}|

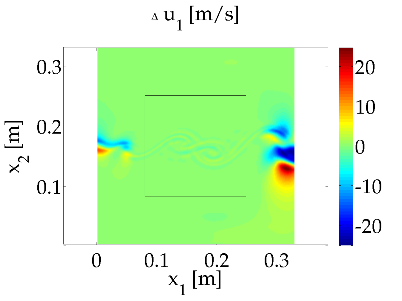
Flow analysis: pressure from PIV



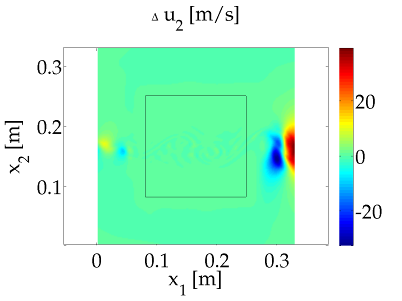
Optimisation of boundary terms



Results

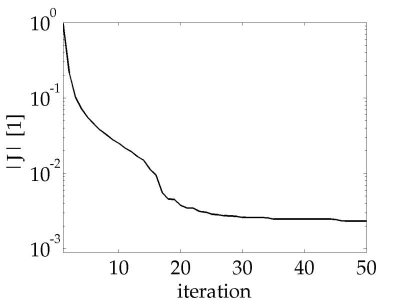


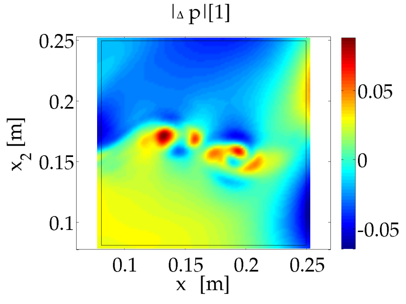
△ u1 [m/s]
0.3
0.2
0.1
x2 [m]
0
0.1
0.2
0.3
x1 [m]
20
10
0
-10
-20



△ u2 [m/s]
0.3
0.2
0.1
x2 [m]
0
0.1
0.2
0.3
x1 [m]
20
0
-20

Results

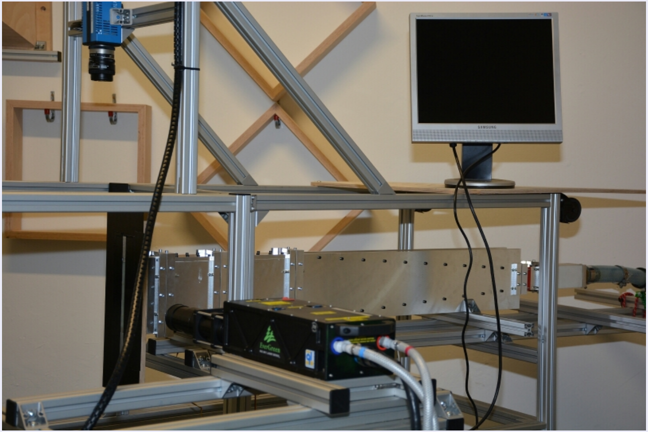




Computation time 5h (single core MatLab)

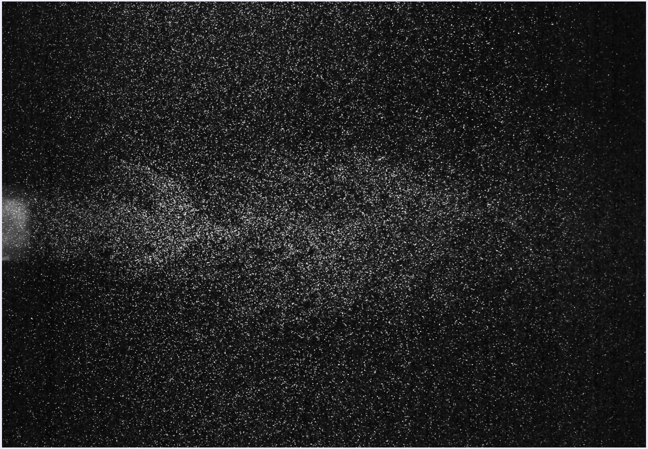
Experiment
Facility

Buildup



High sub-sonic range

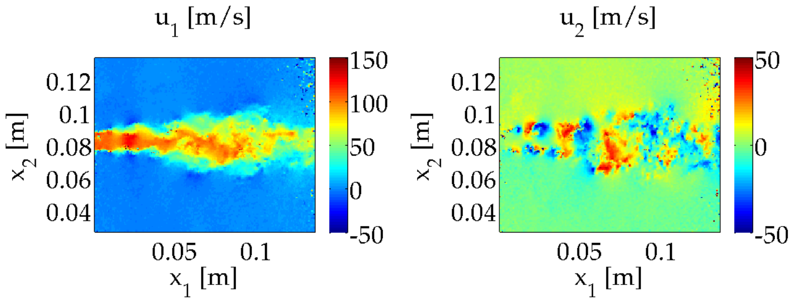
'2D' jet, core velocity $100 ms^{-1}$



Experiment

Measurement

PIV based velocities



Conclusion

Done

- J.Fernandez, R.Wilke,& M.Lemke did a good job so far
- Jet Noise characteristics can be traced back to simple mode interaction
- modes can be identified from real experiments

To do

- Describe Mode dependency on
  - *Re*
  - *Ma*,
- capture bifurcations
- charactersize flow state from measurements

Acknowledgments

The simulations were performed on the national supercomputers Cray XE6 and Cray XC40 at the High Performance Computing Centre Stuttgart (HLRS) under the grant number GCS-ARSI/44027. Typically 8192 cores were used.