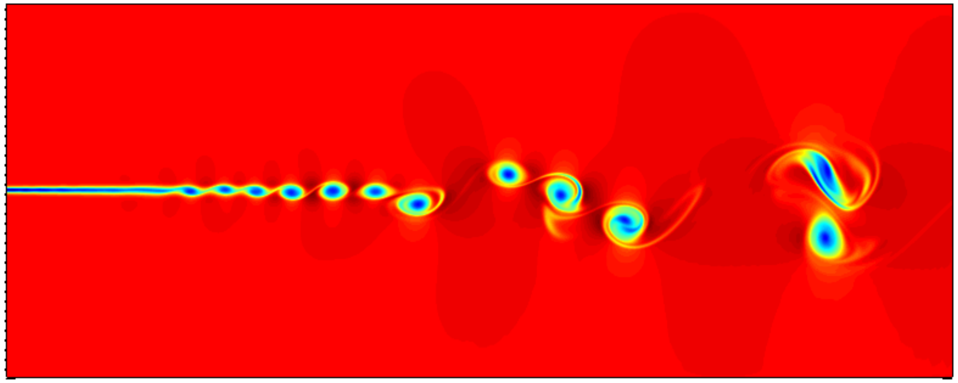
Cluster-based reduced-order modelling (CROM) of a mixing layer

Andrey Markov (1856-1922)



E. Kaiser, B. R. Noack, L. Cordier and A. Spohn .... *Institute PPRIME*

M. Segond and M. Abel .... *Ambrosys GmbH*

G. Daviller .... *CERFACS*

J. Östh and S. Krajnović .... *Chalmers University*

R. K. Niven .... *ADFA/UNSW*

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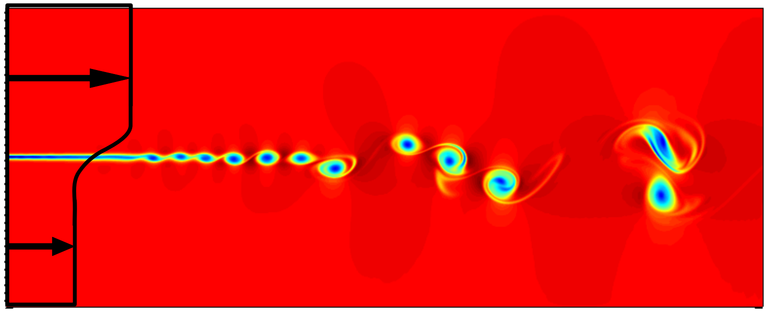
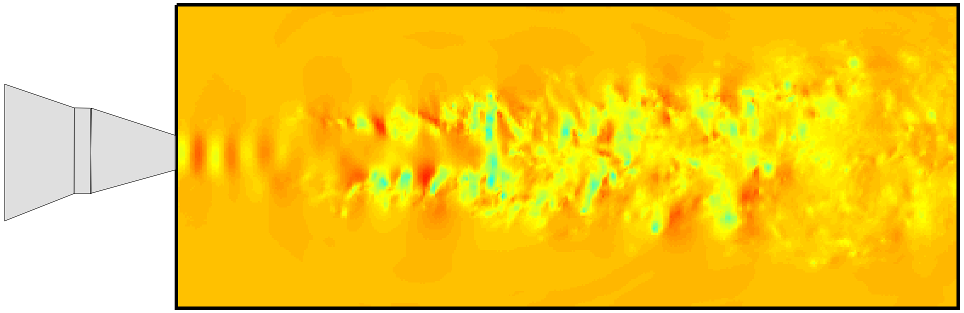
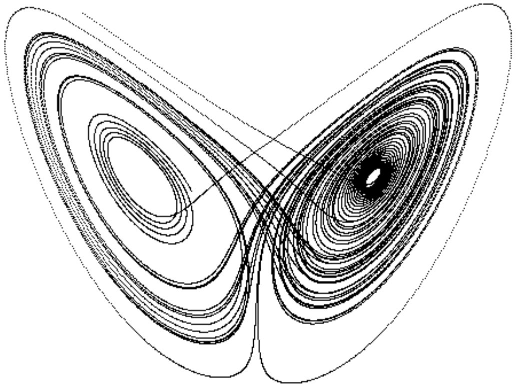
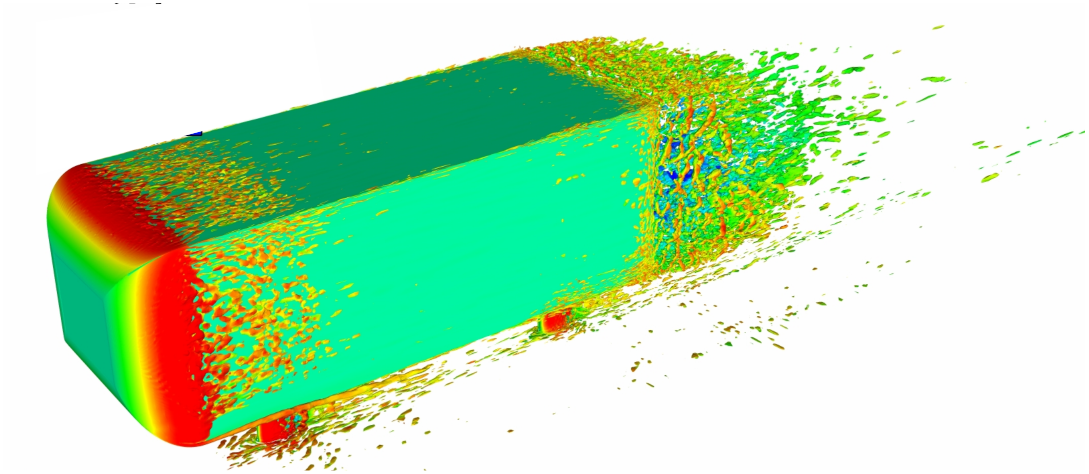
IMFT, Toulouse, 2015-01-14

Motivation









How can one identify physical mechanisms
in an unsupervised manner based on given data?

Reduced-order modelling

Navier-Stokes equation

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla P - \frac{1}{Re} \Delta \boldsymbol{u}$$

POD Galerkin models

$$\frac{\mathrm{d}a_i}{\mathrm{d}t} = c_i + \sum_{j=1}^{N} l_{ij} a_j + \sum_{j,k=1}^{N} q_{ijk} a_j a_k$$

$$\tilde{\boldsymbol{u}}(\boldsymbol{x}, t) = \sum_{i=0}^{N} a_i(t) \boldsymbol{u}_i(\boldsymbol{x}), \; a_0 \equiv 1$$

Liouville equation

$$\partial_t p + \nabla \cdot (\boldsymbol{f} p) = 0$$

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CROM approach

Kaiser et al (2013) JFM Preprint; Burkardt, Gunzburger & Lee (2006); Schneider, Eckhardt & Vollmer (2007)

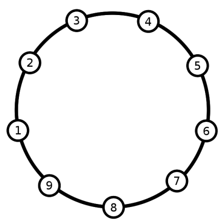
How can one identify physical mechanisms in an unsupervised manner based on given data?

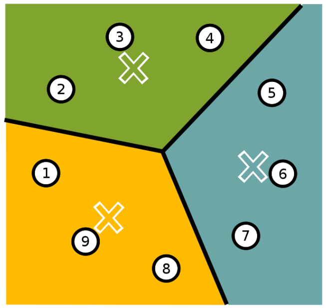
Kinematics

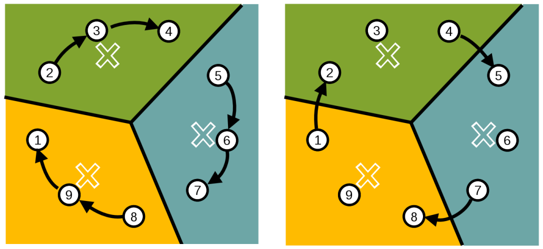
Dynamics

Data → Discretised state space (cluster analysis) → Construct cluster Transition matrix → Markov model

Discrete snapshots of a limit cycle

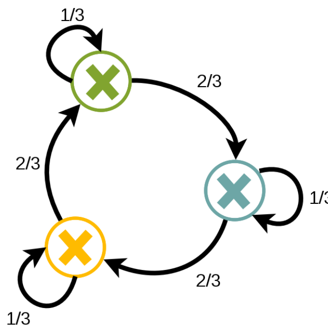






P_{jk}	k=1 1→j	k=2 2→j	k=3 3→j
j=1	2/3	0	1/3
j=2	1/3	2/3	0
j=3	0	1/3	2/3

$$\boldsymbol{p}^l = \boldsymbol{P}^l \boldsymbol{p}^0$$



Time-resolved velocity snapshots

$$\boldsymbol{u}^m = \mathbf{u}(\mathbf{x}, t_m)$$

Cluster analysis (k-means) (Steinhaus 1956, MacQueen 1967)

$$V = \sum_{k=1}^{K} \sum_{\mathbf{u}^m \in \mathcal{C}_k} ||\mathbf{c}_k - \mathbf{u}^m||^2$$

$\rightarrow$ Minimum

Discrete-time Markov model (Time-homogeneous, memoryless)

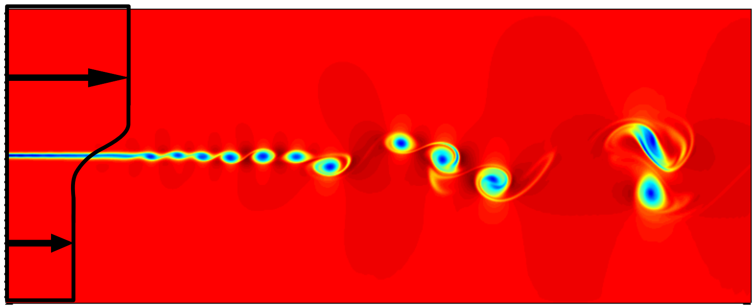
$$\mathbf{p}^l = P^l \mathbf{p}^0 \text{ with } P_{jk} = \frac{n_{jk}}{n_k}$$

→ Refined analysis: Cluster sociology Entropy

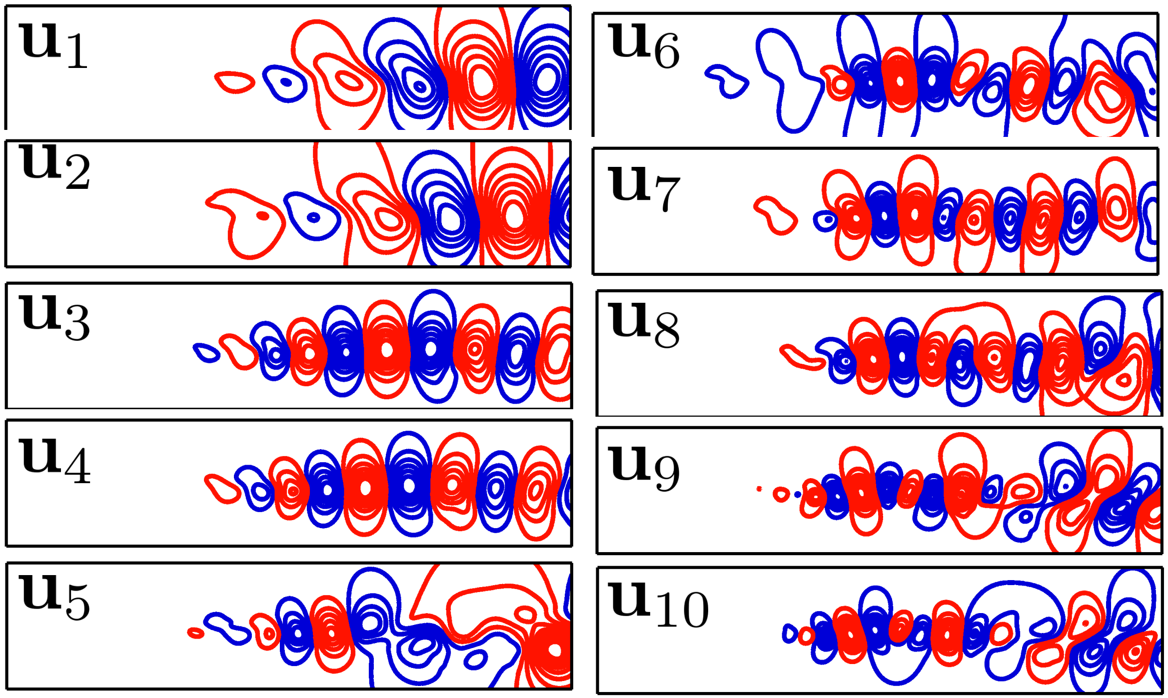
Mixing layer

Daviller (2010); Cavalieri et al. (2011); Noack eta al. (2003); Cordier et. al (2013)

- 2D incompressible mixing layer
- Velocity ratio $\mathrm{r} = \mathrm{U}_1/U_2 = 3$
- Reynolds number $\mathrm{Re} = \mathrm{U}_1\delta_\omega/\nu = 500$
- Finite-difference Navier-Stokes solver (Daviller)
- $\mathrm{M} = 2000$ snapshots



POD Galerkin model



POD expansion:

$$\tilde{\boldsymbol{u}}(\boldsymbol{x},t) = \sum_{i=0}^{N} a_i(t)\boldsymbol{u}_i(\boldsymbol{x}),\ a_0 \equiv 1$$

Galerkin projection:

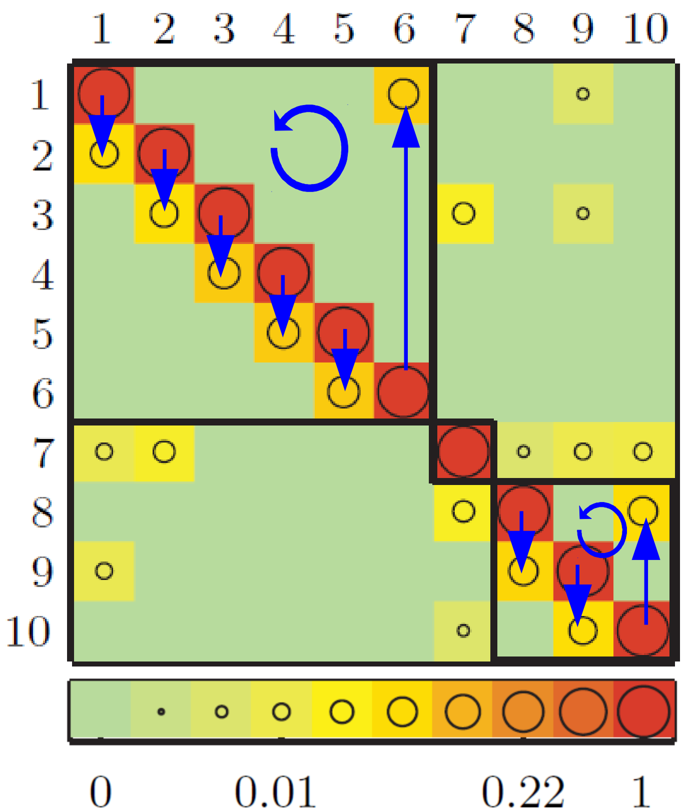
$$\left(\boldsymbol{u}_i, \mathcal{R}\left(\frac{\partial}{\partial t}\tilde{\boldsymbol{u}} - \mathbf{f}(\tilde{\boldsymbol{u}})\right)\right)_\Omega = 0,\ i = 1, \dots, N$$

Dynamical system:

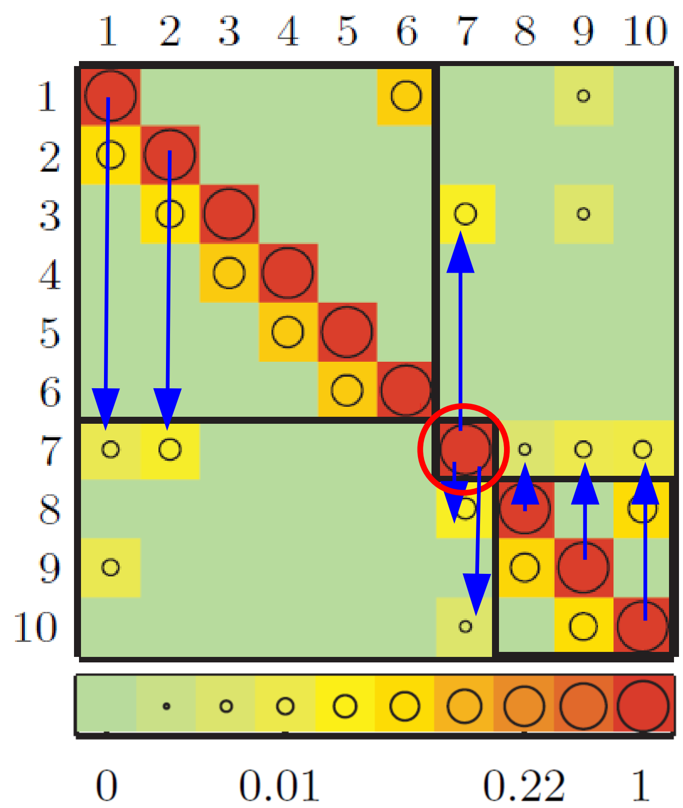
$$\frac{\mathrm{d}a_i}{\mathrm{d}t} = c_i + \sum_{j=1}^{N} l_{ij}a_j + \sum_{j,k=1}^{N} q_{ijk}a_j a_k$$

Markov model of the mixing layer

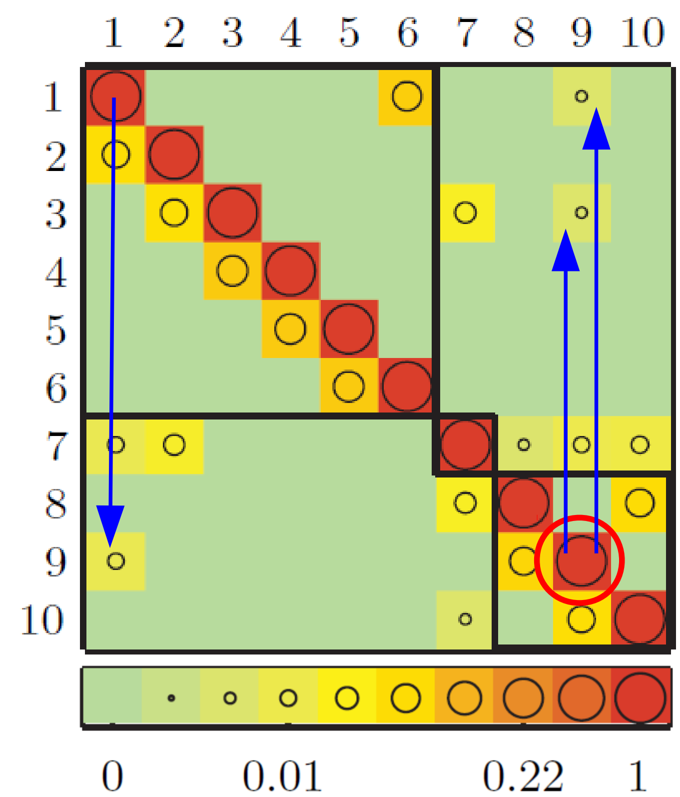
Oscillations:
Intrinsic periodical behaviour is distilled.



Flipper cluster:
Transitions between the two groups are mostly via cluster 7.

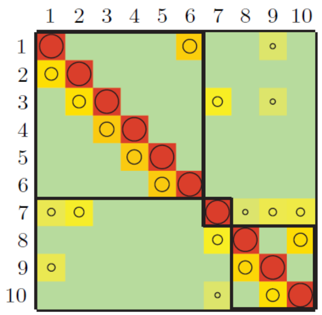


Direct transition:
Transition between the two groups is only possible via cluster 9.

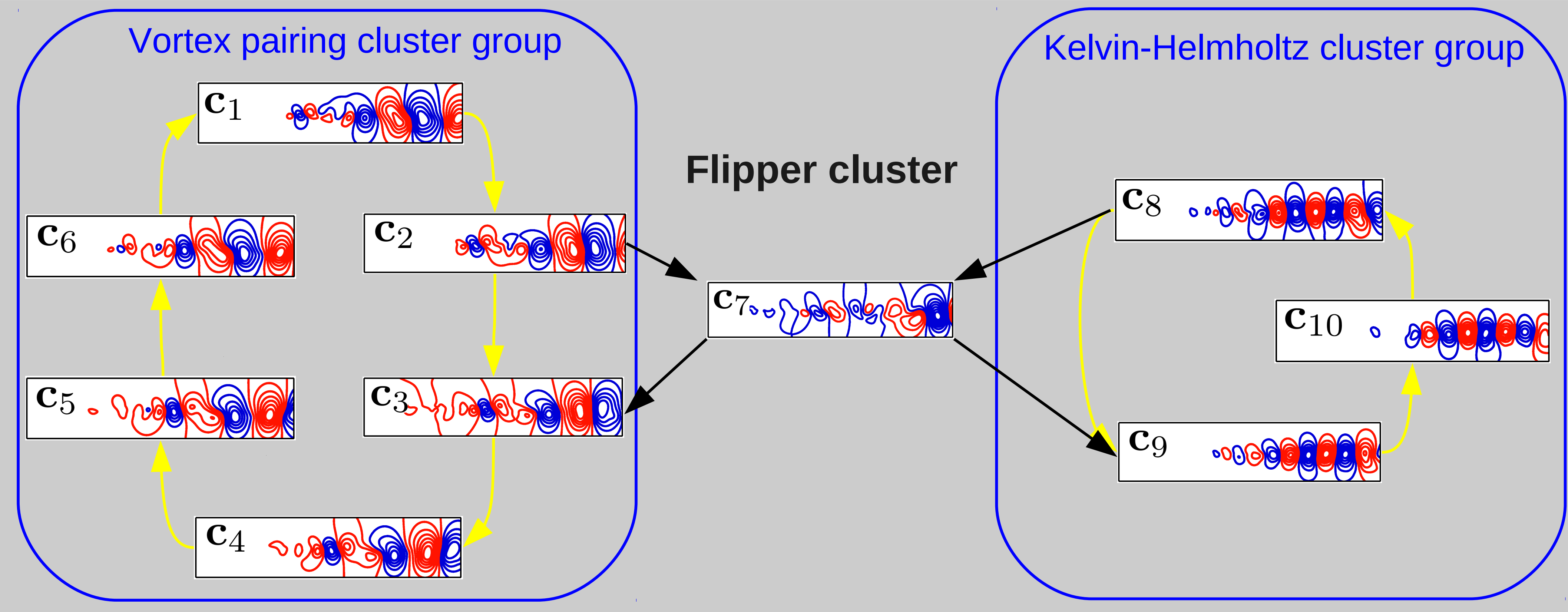


Identification of branching clusters between different regimes.

Mixing layer – Cluster transition model



- Most clusters are 'phase bins'.
- Centroids are aligned with the dynamical evolution of the flow.
- Identification of two shedding regimes.
- Flipper cluster acts as a switch between both shedding regimes.

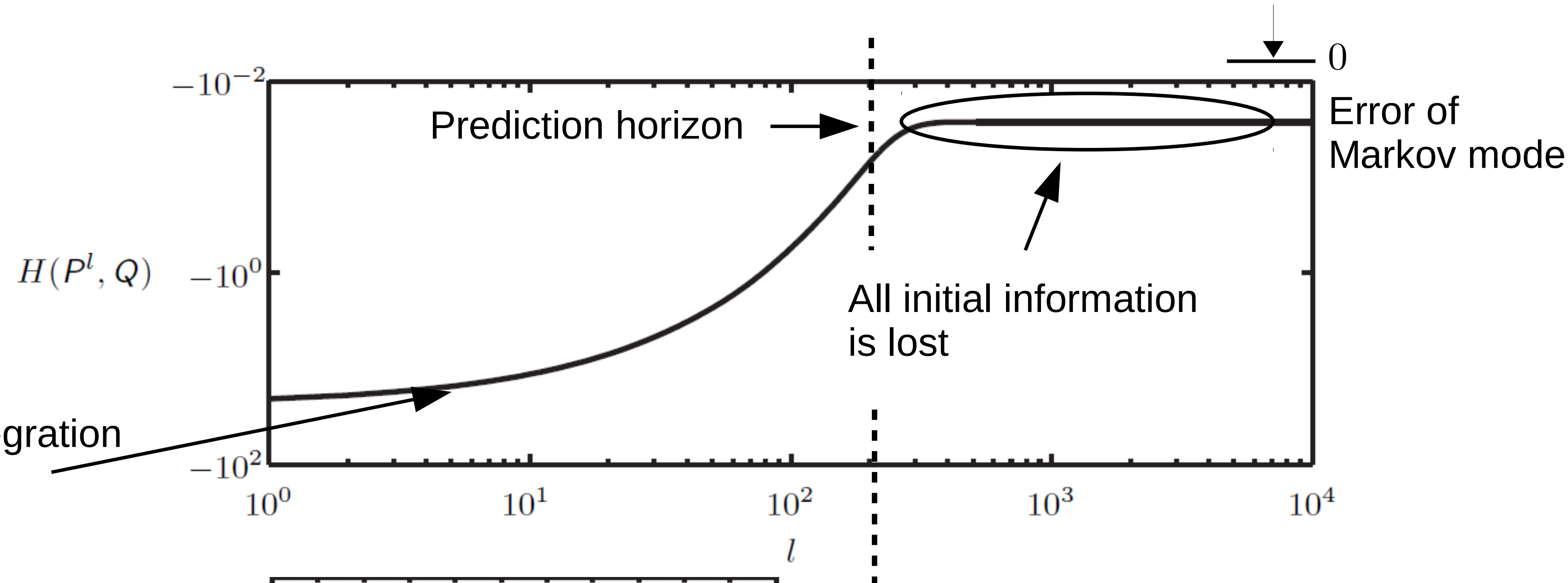


Mixing layer – Attractor properties

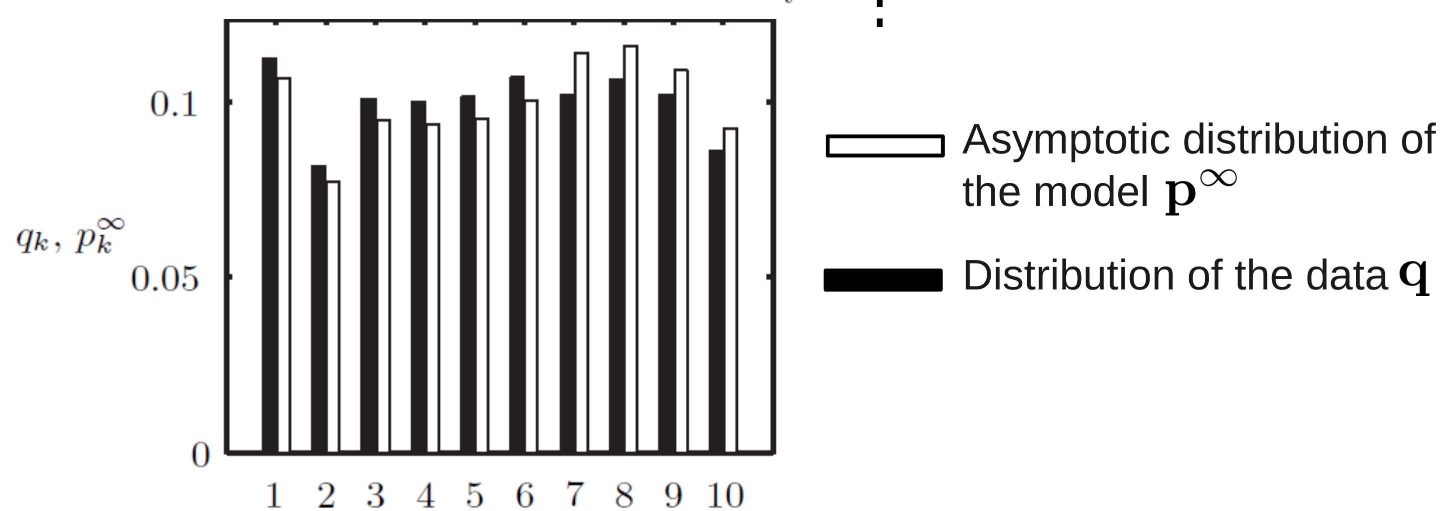
Kullback-Leibler entropy

$$H(P,Q) = -\mathfrak{D}(P,Q) := -\sum_{j=1}^{K}\sum_{k=1}^{K} P_{jk} \ln \frac{P_{jk}}{Q_{jk}}$$

Transients

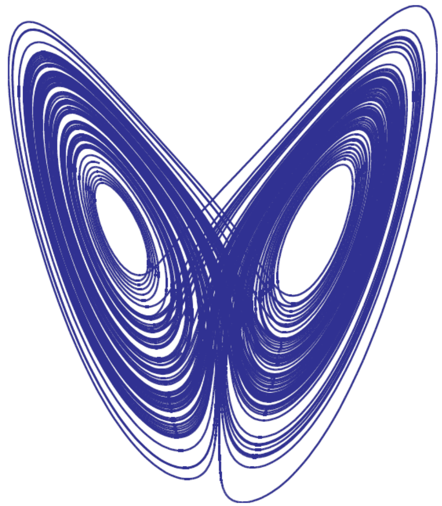


Attractor



Lorenz attractor

Lorenz (1963)



Lorenz equations:

$$\frac{\mathrm{d}x}{dt} = \sigma(x - y)$$

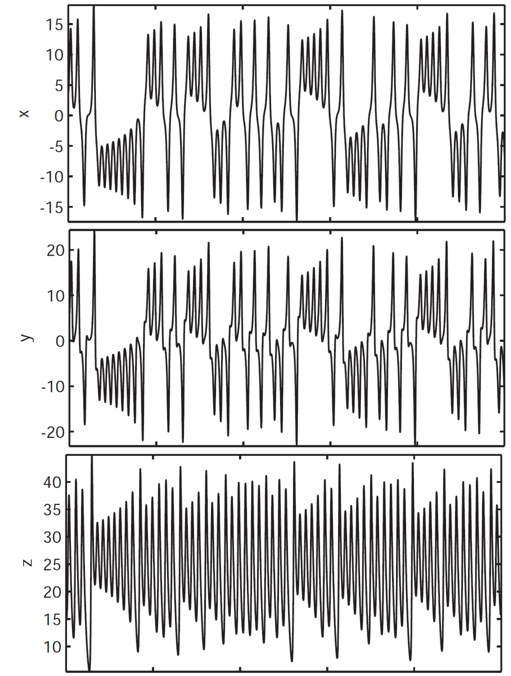
$$\frac{\mathrm{d}y}{dt} = x(\rho - z) - y$$

$$\frac{\mathrm{d}z}{dt} = xz - \beta z$$

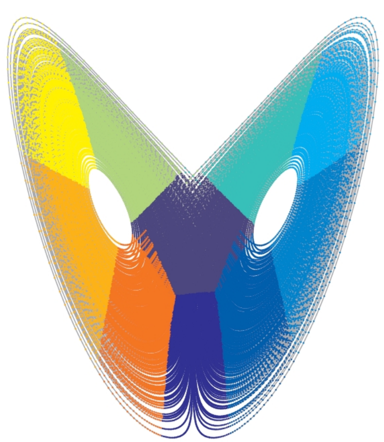
$$\sigma = 10, \quad \beta = 8/3, \; \rho = 28$$

Equidistantly sampled solution:

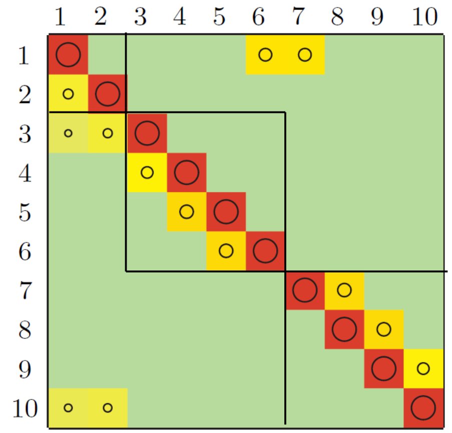
$$\mathbf{x}(\mathrm{t}_m) = (x, y, z)^T(t_m), \; m = 1, 2, \ldots, M$$

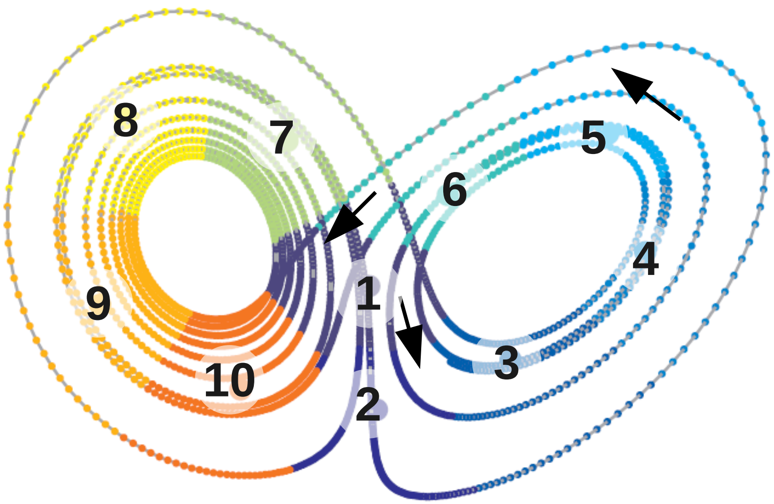


Clustered Lorenz attractor



Transition matrix

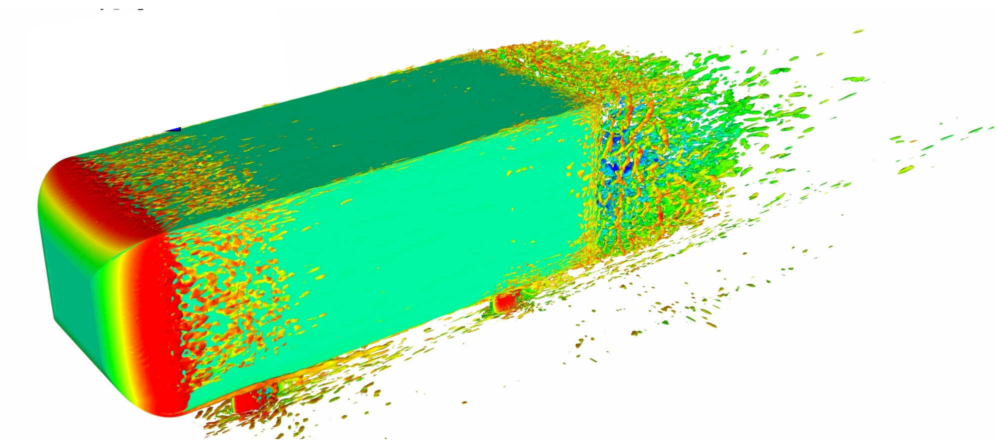




Identification of branching regions and oscillatory cluster groups.

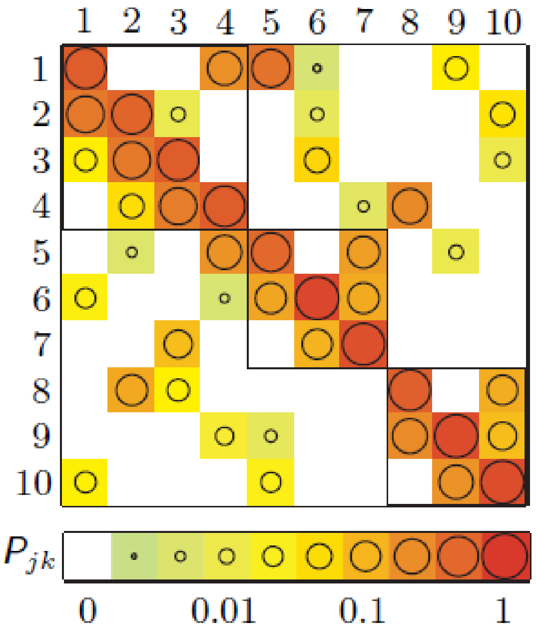
Ahmed body

Oesth, Noack, Krajnovic (2013) JFM

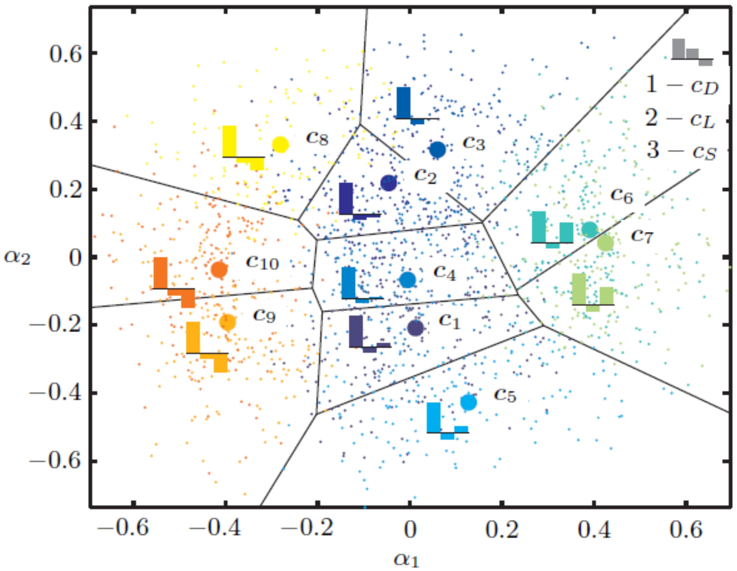


- 3D turbulent bluff body , LES
- Reynolds number $Re = 3 \cdot 10^5$
- $\mathrm{M} = 2000$ snapshots

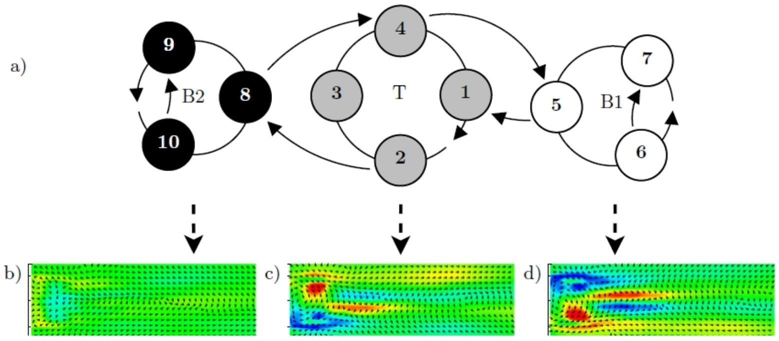
Transition matrix



Forces



Simplified cluster transitions



a) Three cyclic groups: two asymmetric meta-stable states & sym. transition region

b-d) Mean velocity fields of thecluster groups.

Identification of bi-modal states.

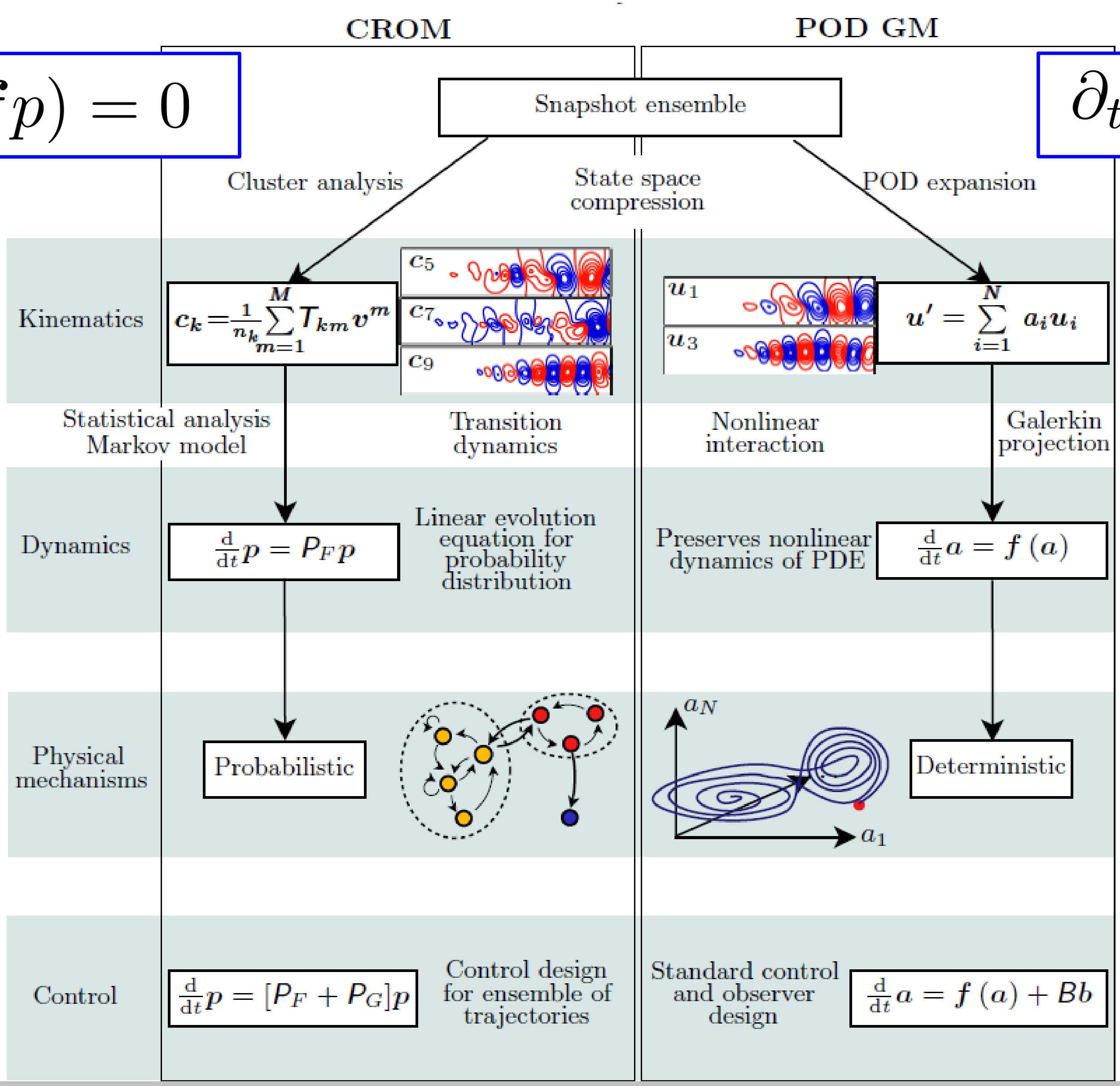
Comparison CROM vs. POD GM

Liouville equation

$$\partial_t p + \nabla \cdot (\boldsymbol{f} p) = 0$$

NSE

$$\partial_t \boldsymbol{u} = \boldsymbol{F}(\boldsymbol{u})$$



Do you want to know more about it? - Visit: www.ClusterModelling.com