

Math circle summer problems

1. In the following problem each player wants to win. There are two piles of 7 stones each. At each turn, a player may take as many stones as he chooses, but only from one of the piles. The loser is the player who cannot move. Determine who can always achieve this goal (i.e. who has the winning strategy) and why.
2. A circle is divided into 6 sectors, and a pawn stands in each of them. At each turn you must move two pawns. They may each only move into a sector bordering their original sector. Is it possible to gather all pawns in one sector when following these rules?
3. Show that for $n \geq 6$ a square can be dissected into n smaller squares, not necessarily all of the same size.
4. On a 5×5 chessboard, place 5 wolves (which can move like queens) and 3 sheep so that all the sheep are safe from the wolves.
5. Three missionaries and three cannibals want to get to the other side of a river. There is a small boat, which can fit only two. To prevent a tragedy, there can never be more cannibals than missionaries together. Can they all cross the river?

More difficult problems

6. Three players play the game "rock-paper-scissors". In every round, each player simultaneously shows one of these shapes. Rock beats scissors, scissors beat paper, while paper beats rock. If in a round exactly two distinct shapes are shown (and thus one of them is shown twice) then 1 point is added to the score of the player(s) who showed the winning shape, otherwise no point is added. After several rounds it occurred that each shape had been shown the same number of times. Prove that the total sum of points at this moment was a multiple of 3.
7. 100 prisoners in separate secluded cells and a prison warden agree to perform the following experiment over a period of time: Each day the warden will select a

prisoner purely at random (it is thus possible that a prisoner may be selected multiple times as the days go by) and lead her to a room that contains nothing but a light bulb hanging from a cord from the center of the ceiling. The prisoner led to the room that day has the option of doing one of the following four things:

- Remain silent and switch the light off if it was on.
- Remain silent and switch the light on if it was off.
- Remain silent and do nothing.
- Make the statement: "All 100 prisoners have now been led to this room."

If the prisoner making a statement is correct, all will go free. If the statement is not correct, all will be executed. The prisoners performing this experiment cannot communicate in any way (except through on/off state of the light bulb) and they do not see who is being led to the room on any day. What strategy can the prisoners all agree on before they play the game to ensure that they will all go free? They know the game will start with the light bulb initially on.

8. 100 mathematicians go on a jungle trip. Cannibals ambush a safari in the jungle and capture them. The cannibals give the men a single chance to escape uneaten. The captives are lined up in order of height, and are tied to stakes. Each person will be assigned either a red hat or a blue hat. No one can see the color of his or her own hat. However, each person is able to see the color of the hat worn by every person in front of him or her. That is, for example, the last sage in line can see the color of the hat on 99 mathematicians in front of him or her; and the first person, which is at the front of the line, cannot see the color of any hat. Beginning with the last person in line, and then moving to the 99th person, the 98th, etc., each will be asked to name the color of his or her own hat. If the color is correctly named, the captive can leave unharmed. Everyone in line is able to hear every response. Before being lined up, the 100 persons are allowed to discuss strategy, with an eye toward developing a plan that will allow as many of them as possible to name the correct color of his or her own hat (and thus survive). Once lined up, each person is allowed only to say "Red" or "Blue" when his or her turn arrives, beginning with the last person in line. Your assignment: Develop a plan that allows saving as many mathematicians as possible from the gruesome fate.