Math Circle 10/8/2016 Meeting: Challenge of the week

Make sure that your solution is correct, complete, and clearly written. You should not expect much credit if your proof refers to a false statement, or even if all your statements are true but you forgot to tell us "why?" It is one of the purposes of the Circle to help you improve your "essay-proof" writing style as well as your logical skills.

Problem 3. Suppose points A and B, and a circle with diameter PQ are positioned as in the figure below. Using only a pencil and a straightedge, construct a perpendicular from A onto PQ and from B onto the continuation of PQ.

Solution: *See the graphical construction of the perpendicular for point A below. To prove that AZ is indeed the perpendicular, note that since PQ is a diameter of the circle, we have $\angle PYQ = \angle PXQ = 90°$, which means that QY and PX are altitudes of the triangle APQ. Since altitudes of any triangle intersect in a single point (called the orthocenter of the triangle), we conclude that the line AZ is indeed perpendicular to PQ. We leave the construction of the perpendicular for the point B (and its proof) as an exercise.*