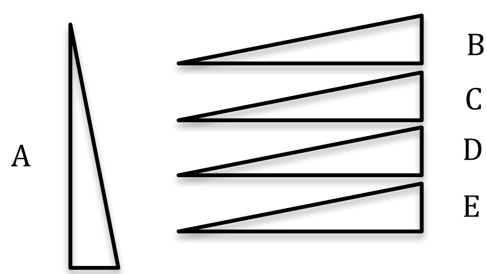
Math Circle 9/24/2016 Meeting: Challenge of the week

Make sure that your solution is correct, complete, and clearly written. You should not expect much credit if your proof refers to a false statement, or even if all your statements are true but you forgot to tell us "why?" It is one of the purposes of the Circle to help you improve your "essay-proof" writing style as well as your logical skills.

Problem 2.

A. There are 5 identical paper triangles on the table. Each can be translated, i.e. moved in any direction parallel to itself (without rotating it). Is it true that regardless of the initial position of the triangles, translating 4 triangles we can always completely cover the remaining one (overlaps are allowed)?

Solution: No. In the following example the four triangles B, C, D, E cannot cover triangle A.



B. There are 5 identical equilateral paper triangles on the table. Each can be translated. Prove that, regardless of the initial position of the triangles, translating 4 triangles we can always completely cover the remaining one (overlaps are allowed).

Solution: The idea is to use the fact that circles are rotationally invariant (as opposed to triangles). Consider the inscribed circle for each equilateral triangle. It is clear that it is a circumscribed circle for the equilateral triangle of the side length equal to a half of the original triangle (see the diagram below). Let us divide triangle A into four congruent triangles A_1, A_2, A_3, and A_4, and let us translate triangles B, C, D, and E so *that their inscribed circles coincide with the circumscribed circles for A_1, A_2, A_3, and A_4. This procedure ensures that we covered triangle A completely.*