*Math Circle 8/27/2016 Meeting: Challenge of the week - **Solution***

Make sure that your solution is correct, complete, and clearly written. You should not expect much credit if your proof refers to a false statement, or even if all your statements are true but you forgot to tell us "why?" It is one of the purposes of the Circle to help you improve your "essay-proof" writing style as well as your logical skills.

Problem 1. In Ms. Sweetmeat's class, each girl gives a candy bar to every student who is shorter in stature than she. On the other hand, each boy in the class gives a candy bar to every pupil taller than he. The students in Ms. Sweetmeat's class are all of the different height.

1. Construct an example of such class where there are three students who get the same number of candy bars, and everyone else gets less.

Height	55″	56″	57″	58″	59″	60″	61″
Gender	B	B	B	B	B	G	B
# of bars	1	2	3	4	5	5	5

2. Prove that whenever there are three students who get more candy bar than all other students in the class, at least one of them is a girl.
Solution: Note that in Part (a) the students with more candy than the rest are neighbors, if we line the students according to their height, with the girl followed by the (shorter) boy. This is not a fluke, as we will now see.

Let's line up kids by their height. Let X < Y < Z be 3 students with most candy (we write Y > X if the student Y is taller than the student X). If X is a girl, there is nothing to prove, so we can assume that X is a boy. We claim that Y is next in line to X and that Y is a girl. This claim implies that at least one of the three students is a girl, which is what has to be shown. To see why our claim holds, let us represent the class configuration as
(Group 1 of students) < X < S < (Group 2 of students), *where S is a (taller) neighbor of the boy X. Suppose first that S is a boy. In this case X gets candy from all boys in Group 1 and all girls in Group 2. On the other hand, the boy S gets candy from all boys in Group 1 and all girls in Group 2* ***and*** *X, so S gets one more candy than X. But this is impossible, as no one in the class has more bars than X. So S has to be a girl. In this case, X gets candy from all boys in Group 1 and all girls in Group B,* ***and*** *S. On the other hand, the girl S gets candy from all boys in Group 1 and all girls in Group 2,* ***and*** *X. That means X and S gets the same number of bars, so S=Y, as claimed.* ■

Exercise: We have shown that if X < Y < Z are students with most candies, and X is a boy, then Y is his tallest neighbor, if we line students by their height, and that Y is a girl. Show that in this case Z is the taller neighbor of Y, and that Z is a boy.