Math Circle 10/1/2016 Meeting: Warm-up problems

(Sources: A Moscow Math Circle: Week-by-Week Problem Sets, Mathematical Circles: (Russian Experience))

Problem 1. The evil king wrote three secret 2-digit numbers a, b, c. A handsome prince must name three numbers X, Y, Z, after which the king will tell him the sum aX+bY+cZ. The prince must then name all three of the king's numbers, or face execution otherwise. Help him out of this dangerous situation.

Problem 2. (a) There are 10 baskets arranged in a circle. Is it possible to arrange several oranges in the baskets so that the difference in the number of oranges in any two adjacent baskets will be 1? (b) What if there are three baskets? (c) What if there are three baskets?

Problem 3. "Andy has more than 1000 books." "No, he has less than 1000 books." "Well, he has to have at least one book." If it is known that only one of these three statements is true, how many books can Andy have?