COUNTING AND COMBINATORICS, GRADES 8-9

BLACKSBURG MATH CIRCLE

The problems below are taken from ”A Decade of the Berkeley Math Circle", Session 2.

Problem 1. A taqueria sells burritos with the following fillings: pork, grilled chicken, chicken mole, and beef. Burritos come either small, medium, or large, with or without cheese, and with or without guacamole. How many different burritos can be ordered?

⚠ *If the thing we are counting is the outcome of a multi-stage process, then the number of outcomes is the product of the number of choices for each stage.*

Problem 2. How many subsets does a set with 8 elements have, including the empty set and the whole set itself?

Problem 3. Given a pool of 30 students, how many ways can we choose a 3-person government consisting of a president, vice-president, and treasurer? You might want to try first the case of 4 rather than 30 students (you can write down all possible outcomes in this case).

⚠ *We denote the number of permutations of n objects taken k at a time by the symbol $P(n,k)$. It is the answer to the question:*

Problem 4. How many different ways can we choose k different things from a set of n objects, where the order of choice matters?

⚠ *Note that $P(n,n) = n!$, pronounced "n factorial", is the product $1 \cdot 2 \cdot \ldots \cdot n$.*

Problem 5. 10 boys and 9 girls sit in a row of 19 seats. How many ways can this be done if
(a) All boys sit next to each other and all girls sit next to each other?
(b) Each child has only neighbors of the opposite sex?

Problem 6. How many ways can you choose a team from 11 people where the team must have at least one person and must have a designated captain?

Problem 7. How many even 3-digit numbers have no repeating digits?

⚠ *Break the outcomes into several separate, mutually exclusive cases, i.e., every possible outcome must belong to exactly one of these cases.*

⚠ *Whenever we partition the outcomes of something into several cases, each requiring different counting methods, we add the number of outcomes in each case to get the total number of outcomes.*

Problem 8. Three different flavors of pie are available, and seven children are each given a slice of pie.

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(a) In how many ways one can do that?
(b) Suppose in addition that at least two children have to get different flavors. In how many ways can this be done?

⚠ *Counting the complement method: We partition the total set of outcomes into the things we are interested in, and the rest (its negation or complement). If the total is easy to count and the negation is easy to count, then we count the complement and our answer is just the difference of the two.*

Problem 9. (a) We roll two dice, one red, one blue. How many different outcomes are possible? What if we have one more green dice?
(b) How many different outcomes can we have if the two dice have the same color (say white)? Write down all possible outcomes.
(c) What would be the answer if the three dice have the same color?

Problem 10. How many 10-bit binary strings (strings of 0's and 1's) have exactly 4 zeroes?

Problem 11. How many ways can 7 dogs consume 10 dog biscuits? The dogs are distinguishable; the biscuits are indistinguishable. Dogs do not share.

Problem 11. How many subsets of the set $\{1, 2, \ldots, 30\}$ have the property that the sum of their elements is greater than 232?