COUNTING AND COMBINATORICS, GRADES 6-7, SESSION 2

BLACKSBURG MATH CIRCLE

Some problems are taken from ”Mathematical Circles (Russian Experience)”, Chapter 2.

If n is a natural number, then $n!$, pronounced “n factorial” , is the product $1 \cdot 2 \cdot \ldots \cdot n$. A permutation of the numbers $1, \ldots, n$ is a sequence of n of these numbers where no two are repeated. There are $n!$ permutations of numbers $1, \ldots, n$.

Problem 1. (a) Calculate $3!, 4!, 5!$;

(b) Calculate $\frac{n!}{(n-1)!}$.

Problem 2. An anagram of a word is a rearrangement (or *permutation*) of the letters to form a different word. In mathematics, and for this problem, we often use ”anagram” to mean any permutation of letters in a word, and therefore will consider ”aaarngm” an anagram of ”anagram”.

(a) How many anagrams/permutations does the ”word” REALSPY have?
(b) How many have RE consecutive ?
(c) REALSPY has a number of “true anagrams”, meaning that the resulting permutation has a meaning in English. One example of such true anagram is PARSLEY. Can you find any other true anagram for REALSPY?

Problem 3. Jenny the Jeweler is trying to make a necklace out of beads. The beads have different colors.

(a) How many different necklaces can she make if she has 3 beads and wants to use all of them?
(b) How many different necklaces can she make if she has 6 beads and wants to use all of them?

Problem 4. At the meeting of the 20 most powerful jedi masters, they all sit around a circular table. In how many ways the seating is possible ? (Two seatings are the same if everyone has the same neighbor on the left, and the same neighbor on the right.)

Problem 5. Santa has 20 different chocolate bars, which he wants to divide into 4 bags. In how many ways can he do this ?

Problem 6. (a) In the Neptunian language, there are 6 letters, and each word has either 5 or 6 different letters, all distinct. If all combinations of letters are permissible, how many words are in the Neptunian dictionary ?

Date: September 17, 2016.

(b) In the Jovian language, there are 6 letters, and each word has exactly 6 different letters, but at least one letter is repeated. If all combinations of letters are permissible, how many words are in the Jovian dictionary ?

Problem 7. In this problem, each group of students will pick up 6 different playing cards.

(a) In how many ways can we arrange the 6 cards?

(b) Take 4 cards. Assume we are playing a game where there are 2 players, and each gets 2 cards. How many hands are possible? Write down all possibilities.

(c) Same question as before, but with 6 cards, and we distribute 3 cards to each player.

(d) What if there are 6 cards and 3 players, each getting 2 cards? Is it still easy to write down all possibilities?

The number of ways to chose an unordered collection of k objects taken from n objects is $\frac{n!}{(n-k)!k!}$. This number is denoted by $\binom{n}{k}$ and it is called "n choose k".

Problem 8. (a) How many different outcomes are there if we throw two dice of the same color ? Write down all possible outcomes.

(b*) What would be the answer if there are three dice of the same color?

Problem 9. Find the number of diagonals of a convex polygon with n edges.

Problem 10.

(a) Find the number of arrangements of the word CIRCLE.

(b) Find the number of arrangements of the word MISSISSIPPI.

(c*) Same question as in (b), but we only count those arrangements where all the S's are consecutive, all the P's are consecutive, and M is before P.

(c) Find the number of arrangements of the word SUPERCALIFRAGILISTICEXPIALIDOCIOUS.

Problem 11.* Kathy wants to buy ice-cream for her 4 teammates in the handball team. She will buy one cup of ice-cream for everyone (including herself). The ice-cream shop has 4 different flavors of ice-cream: Euclid's Lime, Newton Strawberry, Wiles Elliptic Chocolate and Grothendieck Derived Vanilla. How many possible orders can Kathy make?