COUNTING AND COMBINATORICS, GRADES 6-7

BLACKSBURG MATH CIRCLE

Some problems are taken from "Mathematical Circles (Russian Experience)", Chapter 2.

Problem 1. How many three-digit numbers are there?

Problem 2. There are 3 different roads from city A to city B and 4 different roads from city B to city C. In how many ways can someone go from city A to city C passing by city B?

Problem 3.
(a) We roll two dice, one red, one blue. How many different outcomes are possible? What if we have one more green dice?
(b) How many different outcomes can we have if the two dice have the same color (say white)? Write down all possible outcomes.
(c*) What would be the answer if the three dice have the same color?

Problem 4. Aladdin finds out that the password to enter the Cave of Wonders is a word no longer than 4 numbers, using only numbers $1, 2, 3$ (repetitions are allowed). How many possibilities there are for this password ?

Problem 5. Jenny the Jeweler is trying to make a necklace out of beads. The beads have different colors.
(a) How many different necklaces can she make if she has 3 beads and wants to use all of them?
(b) How many different necklaces can she make if she has 6 beads and wants to use all of them?

If n is a natural number, then $n!$, pronounced "n factorial" , is the product $1 \cdot 2 \cdot \ldots \cdot n$.

Problem 6. In this problem, each group of students will pick up 6 different playing cards.
(a) In how many ways can we arrange 3 cards? Verify your answer using the cards, and write down all possibilities.

(b) In how many ways can we arrange 4 cards? (Again, record all possibilities).

(c) Take 4 cards. Assume we are playing a game where there are 2 players, and each gets 2 cards. How many hands are possible? Write down all possibilities.

(d) Same question as before, but with 6 cards, and we distribute 3 cards to each player.

(e) What if there are 6 cards and 3 players, each getting 2 cards? Is it still easy to write down all possibilities?

Date: August 27, 2016.

Problem 7. An anagram of a word is a rearrangement (or *permutation*) of the letters to form a different word. In mathematics, and for this problem, we often use "anagram" to mean any permutation of letters in a word, and therefore will consider "aaarngm" an anagram of "anagram".

(a) How many anagrams/permutations does the "word" REALSPY have?
(b) How many of these permutations have R before S ?
(c) How many have RE consecutive ?
(d) REALSPY has a number of "true anagrams", meaning that the resulting permutation has a meaning in English. One example of such true anagram is PARSLEY. Can you find any other true anagram for REALSPY?

Problem 8. (a) Simplify the expressions $10! \cdot 11$, $n! \cdot (n+1)$;
(b) Calculate

$$\frac{4!}{2!}, \quad \frac{100!}{98!}.$$

(c) Prove that if p is a prime number then $(p-1)!$ is not divisible by p.

Problem 9. A captain and a deputy captain must be elected in a soccer team with 11 players. How many ways are there to do this ?

Problem 10. Find the number of diagonals of a convex polygon with n edges.

The number of permutations, or arrangements, of n distinct objects is denoted by $P(n)$. This number is ...

Problem 11. (a) Find the number of arrangements of the word CIRCLE.
(b) Find the number of arrangements of the word MISSISSIPPI.
(c*) Same question as in (b), but we only count those arrangements where all the S's are consecutive, all the P's are consecutive, and M is before P.
(c) Find the number of arrangements of the word SUPERCALIFRAGILISTICEXPIALIDOCIOUS.

Problem 12. Consider the letters a, b, c, d, e, f. Find the number of 3 letter words using letters $a - f$ such that:
(a) All letters are distinct.
(b) All letters are distinct, and e is one of the letters.
(c*) Letter e is contained in the word, and repeated letters are allowed.

Problem 13.* Kathy wants to buy ice-cream for her 4 teammates in the handball team. She will buy one cup of ice-cream for everyone (including herself). The ice-cream shop has 4 different flavors of ice-cream: Euclid's Lime, Newton Strawberry, Wiles Elliptic Chocolate and Grothendieck Derived Vanilla. How many possible orders can Kathy make?

Problem 14.* In how many ways we can place 9 different rings on the 4 fingers of the right hand (the thumb is excluded)? Keep in mind that the order of the rings on fingers matters.