# BLACKSBURG MATH CIRCLE: SATURDAY, NOVEMBER 14, 2015

## Warm-up Problems

*Choose a few of these problems to work on as you get settled in today. You don't need to complete all of the problems now. Once you've thought about a problem on your own, talk to someone sitting near you about your ideas.*

1. Make a conjecture: What is the formula for the sum of $n$ consecutive odd numbers that starts with 1?
2. Make a conjecture: Which number is larger: a natural number $n$ or $2^n$? Which number is larger: the natural number $n^3$ or $2^n$?
3. Make a conjecture: What is a common divisor for $n^3 - n$, where $n$ is a natural number? For $2^{n+2} + 7^n$?
4. Make a conjecture: Which natural numbers $n$ can be written as a sum of 4's and 5's only? A sum of two primes?
5. (BAMO 2015) There are 7 boxes arranged in a row and numbered 1 through 7. You have a stack of 2015 cards, which you place one by one in the boxes. The first card is placed in box #1, the second in box #2, and so forth up to the seventh card which is placed in box #7. You then start working back in the other direction, placing the eighth card in box #6, the ninth in box #5, up to the thirteenth card being placed in box #1. The fourteenth card is then placed in box #2, and this continues until every card is distributed. What box will the last card be placed in?
6. (BAMO 2015) Members of a parliament participate in various committees. Each committee consists of at least 2 people, and it is known that every two committees have at least one member in common. Prove that it is possible to give each member a colored hat (hats are available in black, white or red) so that every committee contains at least two members with different hat colors.
7. (BAMO 2015) Which number is larger, $A$ or $B$, where
$$A = \frac{1}{2015}\left(1 + \frac{1}{2} + \ldots + \frac{1}{2015}\right), \quad B = \frac{1}{2016}\left(1 + \frac{1}{2} + \ldots + \frac{1}{2016}\right)?$$
8. (BAMO 2014) Amy and Bob play a game. They alternate turns, with Amy going first. At the start of the game, there are 20 cookies on a red plate and 14 on a blue plate. A legal move consists of eating two cookies taken from one plate, or moving one cookie from the red plate to the blue plate (but never from the blue plate to the red plate). The last player to make a legal move wins; in other words, if it is your turn and you cannot make a legal move, you lose, and the other player has won.

   Which player can guarantee that they win no matter what strategy their opponent chooses? Prove that your answer is correct.