

BLACKSBURG MATH CIRCLE: SATURDAY, OCTOBER 24, 2015

WARM-UP PROBLEMS

Choose a few of these problems to work on as you get settled in today. You don't need to complete all of the problems now. Once you've thought about a problem on your own, talk to someone sitting near you about your ideas.

1. Find
 - (a) $(2014 \cdot 2015 \cdot 2016 + 2015^3) \pmod 3$;
 - (b) $9^{100} \pmod 8$.
2. The Dragon of Ignorance has three heads and three tails. You can slay it with the Sword of Knowledge, by chopping off all its heads and all of its tails. With one stroke of the sword, you can chop off one head, two heads, one tail or two tails. But The Dragon of Ignorance is hard to slay! If you chop off one tail, two new tails replace it. If you chop off one head, a new one grows in its place. If you chop off two tails, one new head grows. If you chop off two heads nothing grows. Show how to slay The Dragon of Ignorance. How many chops do you need?
3. Prove that the number $n^3 + 2n$ is divisible by 3 for any natural number n .
4. Cinderella is asked by her stepmother to sort out a bag containing the mixture of rye and barley grains. She did not finish sorting before she went to the ball, so she decided to put labels "Rye", "Barley", and "Mixture" on sorted/unordered bags, and finish sorting when she will be back. Unfortunately, one of her stepsisters got home first, and out of pure spite exchanged labels. The fair godmother warned Cinderella that all bags are now labelled wrong. Thanks to this information, Cinderella managed to figure out how to put the labels right by just examining a single grain from a single bag. How did she do it?
5. Given that p , $p + 10$, and $p + 14$ are prime numbers, find p .
6. Can one place a few dimes on the table so that each coin touches exactly three other coins?
7. Prove that the sum of the squares of five consecutive natural numbers cannot be perfect square.
8. 7 natural numbers are written on a circle. Try to prove that there are at least one pair of adjacent numbers whose sum is even.