

**BLACKSBURG MATH CIRCLE: SATURDAY, SEPTEMBER 19,
2015**

WARM-UP PROBLEMS

Choose a few of these problems to work on as you get settled in today. You don't need to complete all of the problems now. Once you've thought about a problem on your own, talk to someone sitting near you about your ideas.

1. Prove that the product of any three consecutive natural numbers is divisible by 6.
2. Prove that the product of any five consecutive natural numbers is divisible by 120.
3. Find the smallest natural number n such that $n!$ is divisible by 990.
4. What natural numbers have odd number of divisors?
5. Tom multiplied two two-digit numbers. Then he changed all the digits to letters (different letters correspond to different digits). He obtained $AB \times CD = EFFF$. Prove that Tom made mistake somewhere.
6. Can a number with 100 zeroes, 100 ones, and 100 twos be a perfect square?
7. The numbers a and b satisfy $56a = 65b$. Prove that $a + b$ is composite (not prime).
8. Jack tore out several consecutive pages from a book. The number of the first page he tore out was 183, and it is known that the number of the last page he vandalized has the same digits as the first one, in a different order. How many pages did Jack tear out of the book?
9. A 17-digit number is chosen, and its digits are reversed, forming a new number. These two numbers are added together. Show that their sum contains at least one even digit.