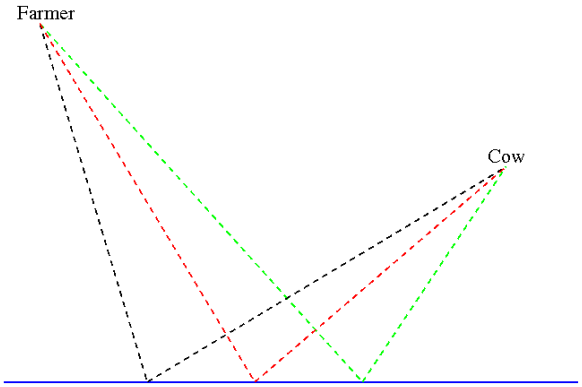
BLACKSBURG MATH CIRCLE: SATURDAY, SEPTEMBER 5, 2015

Warm-up Problems, multiple sources

Choose a few of these problems to work on as you get settled in today. You don't need to complete all of the problems now. Once you've thought about a problem on your own, talk to someone sitting near you about your ideas.

1. A farmer and a cow are on the same side of a straight-line river. (See the figure below, where three possible routes for the farmer are illustrated. The river is the horizontal line.) The farmer has to walk to the river, get water in a bucket, and take it to the cow. What is his shortest path?



2. On a blackboard are written the numbers 1 through 101. At every stage, two are selected, erased from the board, and their difference is added to the list on the board. At any stage, youre free to choose any two numbers. Prove that when the board is reduced to a single number, it cannot be 0.

3. The game of nim. There are two players and they begin with a pile containing 20 pennies. They alternate moves, and for each move, a player can remove 1, 2 or 3 pennies from the pile. When the pile is empty, the game is over and the player who cannot make a move loses. Does the first or the second player win (assume that both use optimal strategies)?

4. Three frogs sit at three vertices of a square. Naturally, they begin to play the game of leapfrog: One of the frogs can jump to a new position which is point reflection about a position of another frog. Can frogs come up with the strategy to bring one of them to the initially unoccupied vertex of the square at the end of the game?