BLACKSBURG MATH CIRCLE: SATURDAY, AUGUST 29, 2015

Warm-up Problems, multiple sources

Choose a few of these problems to work on as you get settled in today. You don't need to complete all of the problems now. Once you've thought about a problem on your own, talk to someone sitting near you about your ideas.

1. Find a two-digit number, the sum of whose digits does not change when the number is multiplied by any one-digit number.

2. Find a way to cut a $3\times$ 9 rectangle into 8 squares.

3. In the following multiplication problem, A,B,C,D,E are different positive integers. Determine their values.

$$\begin{array}{r} A\,B\,C\,D\,E \\ \times\,4 \\ \hline E\,D\,C\,B\,A \end{array}$$

4. Find the smallest integer whose first digit is 7 and which is reduced to 1/3 of its original value when its first digit is transferred to the end. Then find all integers with this property.

5. In equilateral triangle ABC, the point P is on AB so that AP = AB/3 and the point Q is on BC so that BQ=BC/3, and the point R is on CA so that CR=CA/3. The lines CP, AQ, BR enclose a triangle. Find the ratio of the area of this triangle to the area of ABC.

6. Five different numbers are given. By computing all of the different sums of 2 numbers, we get the list 8, 11, 13, 14, 15, 16, 18, 19, 21, where, possibly, some of the numbers in the list have occurred more than once. Find the 5 numbers.

7. Can you arrange the numbers 1,2,...,9 along a circle in such a way that the sum of two neighbors is never divisible by 3, 5, or 7?