

BLACKSBURG MATH CIRCLE: SATURDAY, APRIL 2, 2016

PROBLEMS

1. The evil king wrote three secret 2-digit numbers a, b, c . A handsome prince must name three numbers X, Y, Z , after which the king will tell him the sum $aX + bY + cZ$. The prince must then name all three of the king's numbers, or face execution otherwise. Help him out of this dangerous situation.
2. Can we use the base 1 system?
3. What is the largest 3-digit number in the base system n is equal to?
4. Use the previous problem to prove that if $a = (ABC)_n$ (the 3-digit number in the base system n) is equal to $b = (A'B'C')_n$ (another 3-digit number in the base system n), then $A = A'$, $B = B'$, and $C = C'$.
5. How many different numbers one can construct using at most 3 digits in the n base system?
6. Use the previous two problems to conclude that each integer number smaller than $n^3 - 1$ can be uniquely written in the n base system using at most 3 digits.
7. What is the minimum number of weights which enables us to weigh any integer number of grams of gold from 1 to 100 on a standard balance with two pans? Weights may be placed only on the left pan.
- 8* The same question as in the previous problem, but the weights can be placed on either pan of the balance.