

BLACKSBURG MATH CIRCLE: SATURDAY, AUGUST 22, 2015

WARM-UP PROBLEMS, MULTIPLE SOURCES

Choose a few of these problems to work on as you get settled in today. You don't need to complete all of the problems now. Once you've thought about a problem on your own, talk to someone sitting near you about your ideas.

1. The number 458 is written on a blackboard. It is allowed either to double the number on the blackboard, or to erase its last digit. How can we obtain the number 14 using these operations?
2. Cards with the numbers 7, 8, 9, 4, 5, 6, 1, 2, 3 are laid in a row in the indicated order. It is permitted to choose several consecutive cards and rearrange them in the reverse order. Is it possible to obtain the arrangement 1, 2, 3, 4, 5, 6, 7, 8, 9 after three such operations?
3. Prove that if you reverse the order of the digits in any natural number and subtract the result from the initial number, then the difference is divisible by 9.
4. Which number is greater: 2^{300} or 3^{200} ? Why?
5. The numbers 1, 2, ..., 20 are written on a blackboard. It is permitted to erase any two numbers a and b and write the new number $a + b - 1$. What number can be on the blackboard after 19 such operations?
6. Two players sits at a perfectly round table. Both have an infinite pile of quarters, pennies, dimes, and nickels. The players take turns putting (any) coins down, no overlapping allowed, and coins must rest completely on the table surface. The first player who can't put a coin down loses. Is there a winning strategy for either player?
7. On a blackboard are written the numbers 1 through 100. At every stage, two are selected, erased from the board, and their sum plus product is added to the list on the board. At any stage, you're free to choose any two numbers. When the board is reduced to a single number, what possible values can it have?