

**THE MATH BATTLE OF BLACKSBURG MATH CIRCLE:
SATURDAY, APRIL 9, 2016**

MATH BATTLE PROBLEMS

Problem 1. (Easy) Find a solution of the equation $pi + is + sp = sip$, where each letter represent a different digit between 1 and 9 in the corresponding 2- and 3-digit numbers.

Problem 2. (Medium) You work in a pharmacy, and received a package containing 10 identical bottles of pills. A day later the pharmaceutical company that sent the package informs you that in one of the bottles sent the pills are 10 mg heavier than in the rest of the bottles. What is the minimal number of weightings you can do in order to determine which bottle contains the heavier pills, using only a scale?

Problem 3. (Medium) Given 12 integers, prove that 2 of these integers can be selected such that their difference is divisible by 11.

Problem 4. (More difficult) For a positive integer N , let $\mathcal{S}(N)$ denote the sum of its digits.

- (i) Find the largest value of N such that $N + \mathcal{S}(N) = 2015$;
- (ii) Find the largest value of N such that $N + \mathcal{S}(N) + \mathcal{S}(\mathcal{S}(N)) = 2015$.

Problem 5. (Difficult) In a triangle ABC , $BC = 2AB$. Let D be the midpoint of the side BC and let K be the midpoint of BD . Prove that $AC = 2AK$.

MATH BATTLE RULES

- You will be divided into two teams. You will be given approximately 50 minutes to work on the problems as a team.
- There will be two judges who decide on scores. The decisions of the judges are final.
- Each team selects a captain who serves as spokesperson for the team and also participates in the captain's contest.
- The battle begins with a short question to be answered on the spot. The victor decides whether to begin with the right to challenge or to pass this

right to the other team.

- At each stage of the battle, the team with the right to challenge chooses a problem from those that have yet to be presented and challenges the opposing team to present a solution.
- When challenged, the opposing team may choose to accept the challenge, in which case they present a solution. They may also opt to return the challenge, in which case the original team must attempt to present a solution.
- The team presenting a solution nominates one member who has not yet discussed a problem to provide an explanation. This person has up to five minutes to present as complete a solution to the problem as they are able. Drawing and writing equations is included in this five minutes. The presenter may briefly discuss the problem with their team before stepping to the board, but they may not consult with their team while describing their solution.
- The other team then nominates one member who has not yet discussed a problem to respond to the solution just presented. This person has up to three minutes to point out any flaws or omissions or even supply an alternate solution. The respondent can discuss their rebuttal briefly with their team but may not consult their team while speaking.
- After the presentation and rebuttal, the judges may pose questions to one or both of the speakers.
- The judges then award the points available among the three parties involved: the presenting team, the rebutting team and the judges. Each problem is initially worth 5 points. However, should a team return a challenge, the value of the problem increases to 7 points.
- With one exception, the right to challenge then passes to the next team. Exception: when a team returns a challenge and the original team is unable to make significant progress toward a solution (defined as receiving 3 or fewer points), the right to challenge remains with the original team.