

PROBLEMS ON COUNTING

In all the problems, **probability** of an event (situation) is defined as

$$P = \frac{\text{number of favorable outcomes}}{\text{number of all outcomes}}.$$

1. (Rolling dice) We have two rolling dice, with 6 faces, numbered from 1 to 6.
 - (a) Assuming one dice is red, the other is blue, what is the number of possible outcomes ?
 - (b) Same question as in (a), but assume now that the two dice are both white (hence indistinguishable).
 - (c) If the dice are white, what is the probability to throw a double ? How about throwing a pair with sum divisible by 3 ?

2. How many teams can we form with 10 students ? (We are including the empty team, and the team consisting of all students.)

3. (a) Given a pool of 30 students, how many ways can we choose a 3-person government consisting of a president, vice-president and a treasurer ?
 - (b) What if we allow one student to hold more than one job ?

4. How many ways you can choose a team from 11 people where the team must have at least one person, and it must have a designated captain ?

5. (a) How many ways we can rank 5 wizards for the job of chief wizard ?
 - (b) What about n wizards ?
 - (c) What if we insist that Gandolph is ranked first ? (among the n candidates.)
 - (d) What is the *probability* that Gandolph is ranked first ?
 - (e) What if we only need to rank k of the n candidates ?

The number of obtained in (e) is $P(n, k)$, *the number of arrangements of k different objects from a set of n objects, in such a way that the order matters*. Can you write a formula for $P(n, k)$? If $k = n$ this is called the number of *permutations* of n objects.

6. (a) How many arrangements of the word EUCLID are there ?
 - (b) How many arrangements of the word CIRCLE are there ?
 - (c) Can you count arrangements of MISSISSIPPI ??

7. (a) How many ways are there to choose two pizzas (in any order) out of 10 pizzas ?
 - (b) What if we have any number n of pizzas ? (but still have to choose two of them.)
 - (c) What if we choose k pizzas out of n pizzas ?