Math 2214, Spring 2018, Form A

1. The functions $y_1(t) = e^{t^2}$ and $y_2(t) = e^{t^2+t}$ are both solutions of the differential equation
$$y''y - (y')^2 = 2y^2.$$
Then we can conclude that the following are also solutions EXCEPT

   (a) $2e^{t^2}$.

   (b) $e^{t^2} + e^{t^2+t}$.

   (c) $e^{(t+1)^2}$.

   (d) e^{t^2+3t+2}.

2. The general solution of the system $\mathbf{y}' = A\mathbf{y}$, where
$$A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix},$$
is

   (a) $c_1 e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^t \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

   (b) $c_1 e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

   (c) $c_1 e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^t \begin{pmatrix} t+1 \\ -t \end{pmatrix}$.

   (d) $c_1 e^t \begin{pmatrix} t \\ -t \end{pmatrix} + c_2 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

3. The interval of existence for the solution of the initial value problem $y' = y^2$, $y(3) = 1$ is

   (a) $(-\infty, \infty)$.

   (b) $(3, \infty)$.

   (c) $(-\infty, 4)$.

   (d) $(0, \infty)$.

4. You solve the initial value problem $y' = y^2 + 2t$, $y(1) = 2$, using the Euler method with $h = 0.1$. Then the approximation you find for $y(1.2)$ is

(a) 3.496.

(b) 3.476.

(c) 2.6.

(d) 2.996.

5. A particular solution for the equation $y'' - y' = e^t + 1 + \sin t$ should have the form

(a) $Ae^t + B + C \sin t + D \cos t$.

(b) $Ae^t + Bt + C \sin t$.

(c) $Ae^t + Bt + Ct \sin t + Dt \cos t$.

(d) $Ate^t + Bt + C \sin t + D \cos t$.

6. A stream traverses two lakes flowing downstream, and carrying fresh water as it enters the upper lake. The upper lake contains $4 * 10^9$ gallons of water, and the lower lake containss $2 * 10^9$ gallons of water. The flow rate of the stream is the same at all points and is $4 * 10^6$ gallons per day. A factory situated at the upper lake releases a pollutant at a rate of 200 lbs. per day. Let $Q_1(t)$ and $Q_2(t)$ be the amount, in pounds, of pollutant in the upper and lower lakes, respectively, where time t is measured in days. Assuming that each lake is well mixed, Q_1 and Q_2 obey the system

(a) $Q_1' = 4 * 10^6(200 - Q_1)$, $Q_2' = 4 * 10^6(Q_1 - Q_2)$.

(b) $Q_1' = 200 - Q_1/1000$, $Q_2' = Q_1/1000 - Q_2/500$.

(c) $Q_1' = 200 - 4 * 10^6 Q_1$, $Q_2' = 4 * 10^6(Q_1 - Q_2)$.

(d) $Q_1' = 200t - 4 * 10^6 Q_1$, $Q_2' = 4 * 10^6(Q_1 - Q_2)$.

7. The system

$$x' = 2(x - y)y,$$

$$y' = x + y - 2,$$

has an equilbrium point at (1,1). This equilibrium point is a(n)

(a) unstable node.

(b) center.

(c) saddle.

(d) unstable focus.

8. If $x' = \tan x$, and $x(0) = \pi/3$, then $x(0.1)$ is

(a) $\pi e^{0.1}/3$.

(b) $\arcsin(\sqrt{3}e^{0.1}/2)$.

(c) $\pi e^{-0.1}/3$.

(d) $\arccos(e^{-0.1}/4)$.

9. Consider the system

$$x' = -x + ay,$$

$$y' = -x - y.$$

All solutions of this system approach $(0, 0)$ for $t \to \infty$ if and only if

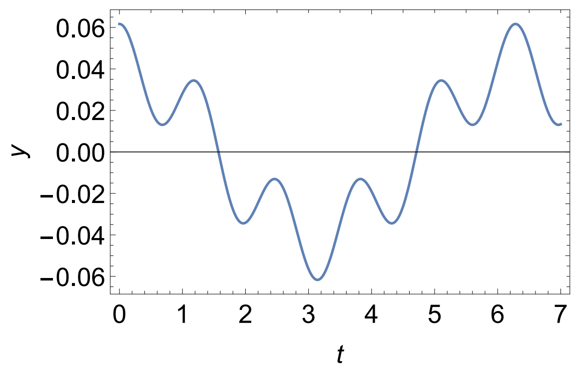
(a) $a < 0$.

(b) $a > -1$.

(c) $a > 0$.

(d) $a < 1$.

10. The following plot shows a solution to the equation $y'' + 25y = \cos(\omega t)$.



The value of ω is

(a) 4.7.
(b) 20.
(c) 5.
(d) 1.

11. A nonlinear system is given by

$$x' = y^2 - xy.$$

$$y' = x^3y^2 - x.$$

The number of equilibrium points is

(a) three.
(b) five.
(c) two.
(d) four.

12. The general solution of the equation $t^2y'' + 2ty' = 0$ is $y = c_1 + c_2/t$. You use the variation of parameters method to look for a solution of $t^2y'' + 2ty' = \sin t$ in the form $y = u_1(t) + u_2(t)/t$. Then u_1 and u_2 should satisfy the system

   (a) $u_1 + u_2/t = 0$, $u_1' - u_2'/t^2 = \sin t$.

   (b) $u_1' + u_2'/t = \sin t$, $u_1' - u_2'/t^2 = 0$.

   (c) $u_1' + u_2'/t = 0$, $u_2' = -\sin t$.

   (d) $u_1' + u_2'/t = 0$, $u_2' = -t^2 \sin t$.