Math 2214, Spring 2016, Form A

1. A matrix is given by
$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}.$$
Then the exponential matrix e^A is

   (a)
$$\begin{pmatrix} e & 0 \\ 2e & e \end{pmatrix}.$$

   (b)
$$\begin{pmatrix} e & 0 \\ e^2 & e \end{pmatrix}.$$

   (c)
$$\begin{pmatrix} e & 1 \\ e^2 & e \end{pmatrix}.$$

   (d)
$$\begin{pmatrix} e & 0 \\ 2 & e \end{pmatrix}.$$

2. The general solution of the system $\mathbf{y}' = A\mathbf{y}$, where
$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
is

   (a) $c_1 \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}.$

   (b) $c_1 e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} -1 \\ -1 \end{pmatrix}.$

   (c) $c_1 \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}.$

   (d) $c_1 e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$

3. The system

$$x' = -x + y, \qquad y' = x(x + y - 2)$$

has a stationary point at $x = y = 1$. The matrix of linearization at this point is

(a)
$$\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$$
.

(b)
$$\begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}$$
.

(c)
$$\begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$$
.

(d)
$$\begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$$
.

4. Suggest a trial form for the particular solution to the following equation

$$y'' + 4y = e^{4t} + \cos(2t) + \sin(4t).$$

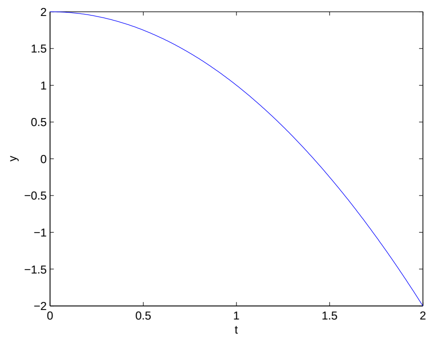
(a) $y_P(t) = Ae^{4t} + B\cos(2t) + C\sin(2t) + Dt\cos(4t) + Et\sin(4t)$.

(b) $y_P(t) = Ae^{4t} + Bt\cos(2t) + Ct\sin(2t) + D\cos(4t) + E\sin(4t)$.

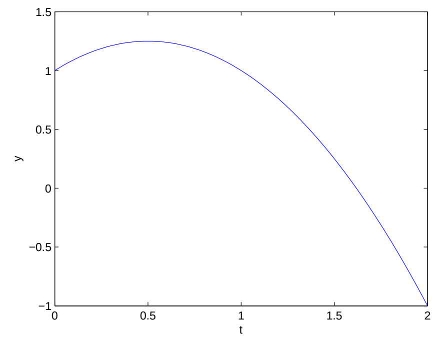
(c) $y_P(t) = Ae^{4t} + B\cos(2t) + C\sin(4t)$.

(d) $y_P(t) = Ate^{4t} + B\cos(2t) + C\sin(2t) + D\cos(4t) + E\sin(4t)$.

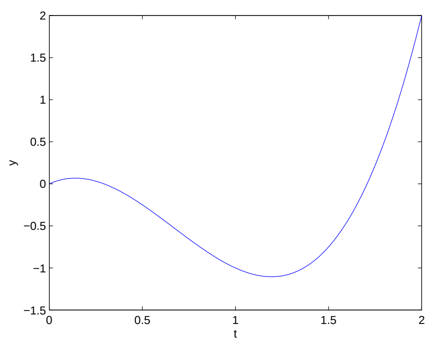
5. Only one of the following graphs shows a solution of the differential equation $y'' = y$. Which is it?



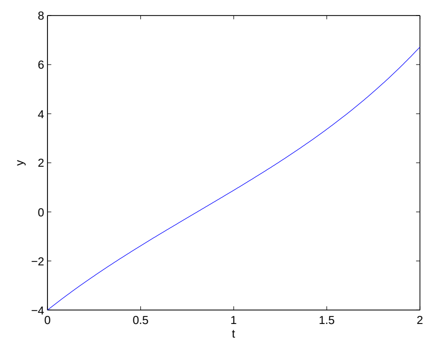
(a)



(b)



(c)



(d)

(a) (a)

(b) (b)

(c) (c)

(d) (d)

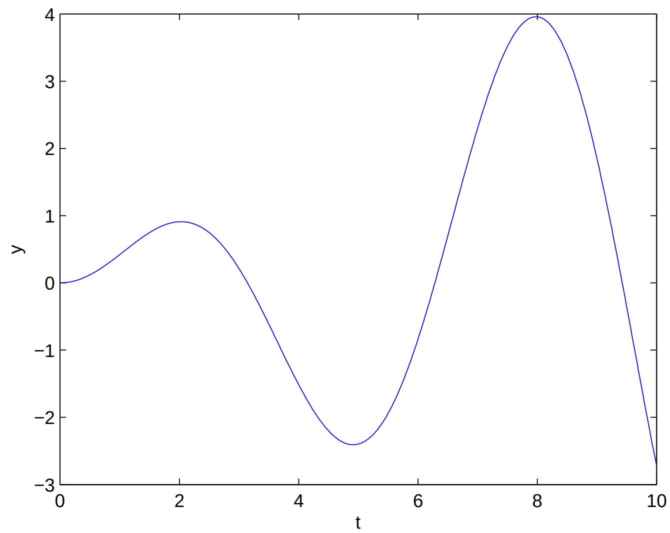
6. The following plot shows a solution of a differential equation. The differential equation is

(a) $y'' + y = \cos(2t)$.

(b) $y'' - 10y' + y = 0$.

(c) $y'' + y = \cos t$.

(d) $y'' + y = t$.



7. You use the Euler method with a step size of 0.1 to solve the initial value problem

$$x' = 2x + y^2, \qquad y' = x + t, \qquad x(0) = 2, \qquad y(0) = 1.$$

Then your approximation for $x(0.2)$ is

(a) 4.16.

(b) 3.144.

(c) 1.46.

(d) 2.5.

8. A particular solution of

$$ty'' + y' = \frac{\ln t}{t}$$

is given by

(a) $5 \ln t/t + 8 \ln t/t^2$.

(b) $2 \ln t/t + 3(\ln t)^2$.

(c) $(\ln t)^3/6$.

(d) $4 \ln t/t^2$.

9. The general solution of the equation

$$y' + \frac{1}{t+1} y = t$$

is

(a) $y(t) = at + b + \frac{C}{t+1}$.

(b) $y(t) = \frac{t^2}{2} + \frac{C}{t+1}$.

(c) $y = \frac{2t^3+3t^2}{6(t+1)} + \frac{C}{t+1}$.

(d) $y(t) = \frac{(t+1)^2}{3} - \frac{t+1}{2}$.

10. The solution of the initial value problem

$$t(1+t)y'' + (\tan t)y' = \ln(2-2t), \qquad y(-1/2) = 2, \qquad y'(-1/2) = -1,$$

is guaranteed to exist on the interval

(a) $(-1/2, 1)$.

(b) $(-\pi/2, \pi/2)$.

(c) $(-\pi/2, 1)$.

(d) $(-1, 0)$.

11. For the system $x' = y$, $y' = -y + x$, the origin is a

(a) saddle point.

(b) unstable node.

(c) stable node.

(d) stable focus.

12. The general solution of the equation $y^{(5)} - y' = 0$ is

(a) $y = a_1 + a_2 e^t + a_3 t e^t + a_4 e^{-t} + a_5 t e^{-t}$.

(b) $y = a_1 + a_2 e^t$.

(c) $y = a_1 + a_2 e^t + a_3 t e^t + a_4 t^2 e^t + a_5 t^3 e^t$.

(d) $y = a_1 + a_2 e^t + a_3 e^{-t} + a_4 \cos t + a_5 \sin t$.