Math 2214, Spring 2015, Form A

1. A nonlinear system is given by

$$x_1' = x_1^2 x_2 - x_1.$$

$$x_2' = x_2 x_1 - x_2^2.$$

The matrix of the linearization at the point $(1, 1)$ is

(a) $\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$.

(b) $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

(c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(d) $\begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix}$.

2. A particular solution of the equation $y''' - y = e^t + \sin t$ should have the form

(a) $ae^t + b\sin(t)$.

(b) $ate^t + be^t + c\sin t$.

(c) $ate^t + b\sin t + c\cos t$.

(d) $ate^t + bt\cos(t) + ct\sin t$.

3. The solution of the initial value problem

$$y_1' = (t-3)y_2 + 1/\cos t, \qquad y_2' = t^3, \qquad y_1(2) = 0, \qquad y_2(2) = 2.5,$$

is guaranteed to exist on the interval

(a) $(-\pi/2, \pi/2)$.

(b) $(\pi/2, 3)$.

(c) $(2, 3)$.

(d) $(\pi/2, 3\pi/2)$.

4. The general solution of the system $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{pmatrix} 2 & 2 \\ -2 & 2 \end{pmatrix},$$

is

(a) $c_1 e^{2t} \begin{pmatrix} \sin(2t) \\ -\sin 2t \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} \cos(2t) \\ \cos(2t) \end{pmatrix}$.

(b) $c_1 e^{2t} \begin{pmatrix} \cos(2t) \\ -\sin 2t \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} \sin(2t) \\ \cos(2t) \end{pmatrix}$.

(c) $c_1 \begin{pmatrix} \cos(4t) \\ -\sin 4t \end{pmatrix} + c_2 \begin{pmatrix} \sin(4t) \\ \cos(4t) \end{pmatrix}$.

(d) $c_1 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

5. Which of the following equations is linear?

(1) $y'' + \sin y = 0$.

(2) $y''' - y'' + y^2 = 0$.

(3) $y'/y = t^2 \sin t$.

(4) $t^5 y'' = \cos t/y$.

(a) (1).

(b) (3).

(c) (2).

(d) (4).

6. Which of the following is a solution of the equation $y'' - y = e^t$?

(a) $t^2 e^t/2$.

(b) $(2t^2 + 1)e^t$.

(c) $(t + 2)e^t/2$.

(d) $t^2 e^t$.

7. The solution of the initial value problem $y' = y^2 - 4y + 3$, $y(0) = 2$ will

   (a) eventually become negative.

   (b) converge to 3 as $t \to \infty$.

   (c) become infinite in finite time.

   (d) converge to 1 as $t \to \infty$.

8. For the system

$$x' = -x + ay,$$
$$y' = 2x - y,$$

   the origin is asymptotically stable if

   (a) $a < 1$.

   (b) $a > 0$.

   (c) $a < 0$.

   (d) $a < 1/2$.

9. If $x' = e^{-x}$, and $x(0) = 5$, then $x(1)$ is

   (a) $5 + e^{-1}$.

   (b) $\ln(e^5 + 1)$.

   (c) $\ln 6$.

   (d) 6.

10. A disposal facility for radioactive waste accepts 200 g of a radioactive substance per day. The half-life of the material is 4 days. If the facility starts out empty, what is the amount of radioactive material in grams which is stored at the facility after 100 days?

   (a) 20000.
   (b) $800(1 - 2^{-25})/\ln 2$.
   (c) $200e^{-25}$.
   (d) $20000 * 2^{-25}$.

11. You solve the initial value problem $y_1' = y_2 + t$, $y_2' = y_1 + 1$, $y_1(0) = 1$, $y_2(0) = 3$, using the Euler method with $h = 0.1$. Then the approximation you find for $y(0.2)$ is

   (a) $(1.6, 3.4)$.
   (b) $(1.65, 3.431)$.
   (c) $(1.62, 3.43)$.
   (d) $(1.63, 3.43)$.

12. A fundamental matrix for the system

$$x' = -x + y,$$

$$y' = -y.$$

   is given by:

   (a) $\begin{pmatrix} te^{-t} & e^{-t} \\ 0 & e^{-t} \end{pmatrix}$.
   (b) $\begin{pmatrix} te^{-t} & 0 \\ e^{-t} & e^{-t} \end{pmatrix}$.
   (c) $\begin{pmatrix} e^{-t} & te^{-t} \\ 0 & e^{-t} \end{pmatrix}$.
   (d) $\begin{pmatrix} e^{-t} & te^{-t} \\ te^{-t} & e^{-t} \end{pmatrix}$.