Math 2214, Fall 2017, Form A

1. Let y be the solution of the system

$$y' = \begin{pmatrix} 3 & 6 \\ 1 & -2 \end{pmatrix} y, \quad y(0) = \begin{pmatrix} a \\ 1 \end{pmatrix}.$$

   Then y tends to zero for $t \to \infty$

   (a) if $a = -1$.
   (b) if $a = 0$.
   (c) for no value of a.
   (d) if $a = 6$.

2. If $x' = 1 + x^2$, and $x(1) = 0$, then $x(5/3)$ is

   (a) $\tan(2/3)$.
   (b) $\sqrt{\exp(2/3) - 1}$.
   (c) 1.
   (d) 0.

3. A pond contains 1 million gallons of water. Every day, 10000 gallons of water flow into the pond. Also, every day 1000 gallons of water evaporate from the pond's surface, and 9000 gallons flow out into the creek downstream. A factory upstream of the pond releases a pollutant at the rate of 100 lbs a day. Let Q be the amount of pollutant in the pond, measured in lbs, and let time be measured in days. Assume the pollutant in the pond is well-mixed. Then the equation satisfied by Q is

   (a) $Q' = 100 - 0.01Q$.
   (b) $Q' = 1000000 - 10000Q$.
   (c) $Q' = 1000000 - 0.009Q$.
   (d) $Q' = 100 - 0.009Q$.

4. Which is an appropriate trial form for the non-homogeneous equation

$$y'' + 2y' + 5y = e^t \cos(2t)$$

(a) $y_P(t) = Ate^t \sin(2t) + Bte^t \cos(2t)$.

(b) $y_P(t) = Ae^t \cos(2t)$.

(c) $y_P(t) = Ate^t \cos(2t)$.

(d) $y_P(t) = Ae^t \sin(2t) + Be^t \cos(2t)$.

5. What is the integrating factor for the differential equation

$$e^{-t}y' - te^{-t}y = \cos(t)?$$

(a) $\mu(t) = e^{\left(te^{-t}+e^{-t}\right)}$.

(b) $\mu(t) = te^{-t} + e^{-t}$.

(c) $\mu(t) = e^{-t^2/2}$.

(d) $\mu(t) = e^{t^2/2}$.

6. Let y be the solution of the initial value problem $y'' - y = e^t$, $y(0) = y'(0) = 0$. Then $y(1)$ is

(a) $e/2 + 1/(4e)$.

(b) $e/4 - 1/(4e)$.

(c) $e/4 + 1/(4e)$.

(d) $e/2 - 1/(4e)$.

7. You solve the initial value problem $y' = t + y$, $y(1) = 1$ using the Euler method with $h = 0.05$. Then the approximation you find for $y(1.1)$ is

(a) 1.2175.

(b) 1.2075.

(c) 1.21.

(d) 1.21263.

8. Consider a 3×3 linear system $\mathbf{y}' = A\mathbf{y}$. Suppose the matrix A has an eigenvalue $\lambda = 3 + 2i$ with eigenvector $\begin{pmatrix} 1+2i \\ 1-2i \\ 1 \end{pmatrix}$. Select a pair of real valued solutions that we can obtain with this information.

$$\textbf{i:}\ e^{3t}\begin{bmatrix} \cos(2t) + 2\sin(2t) \\ \sin(2t) - 2\cos(2t) \\ \cos(2t) \end{bmatrix} \quad \textbf{ii:}\ e^{3t}\begin{bmatrix} \cos(2t) - 2\sin(2t) \\ \cos(2t) + 2\sin(2t) \\ \cos(2t) \end{bmatrix}$$

$$\textbf{iii:}\ e^{3t}\begin{bmatrix} 2\cos(2t) + \sin(2t) \\ \sin(2t) - 2\cos(2t) \\ \sin(2t) \end{bmatrix} \quad \textbf{iv:}\ e^{3t}\begin{bmatrix} \cos(2t) - 2\sin(2t) \\ \cos(2t) + 2\sin(2t) \\ \sin(2t) \end{bmatrix}$$

(a) **i** and **iii**.

(b) **i** and **iv**.

(c) **ii** and **iii**.

(d) **ii** and **iv**.

9. Let $f(t, y)$ be a continuously differentiable function for all values of t and y. You are told that $y = 1$ is a solution of the differential equation $y' = f(t, y)$. Which of the following can not also be a solution?

(a) $y = 0$.

(b) $y = t^2 + 5$.

(c) $y = e^t + 2$.

(d) $y = \cos t$.

10. A nonlinear system is given by

$$x_1' = x_1(x_2^2 - 1).$$

$$x_2' = (x_1 + x_2)(3x_1 - 1).$$

The number of equilibrium points of this system is

(a) Four.

(b) Three

(c) Six.

(d) Five.

11. The matrix A is given by

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

A fundamental matrix for the system $y' = Ay$ is

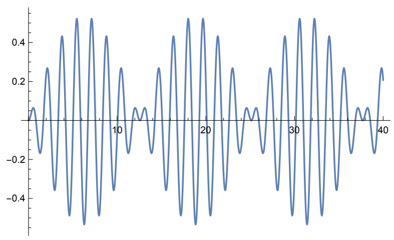
(a) $\begin{pmatrix} 0 & 0 \\ e^t & e^t \end{pmatrix}$.

(b) $\begin{pmatrix} e^t & 0 \\ 1 & e^t \end{pmatrix}$.

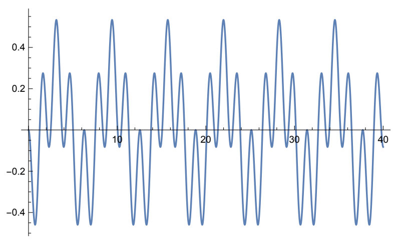
(c) $\begin{pmatrix} te^t & 0 \\ e^t & te^t \end{pmatrix}$.

(d) $\begin{pmatrix} e^t & 0 \\ te^t & e^t \end{pmatrix}$.

12. Which of the following graphs shows the solution of $y''+16y = \cos(3.5t)$, $y(0) = y'(0) = 0$?



(a)



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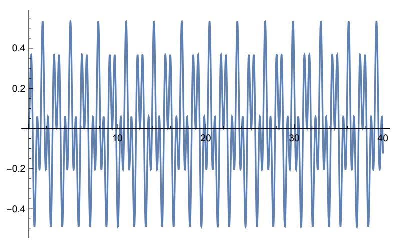

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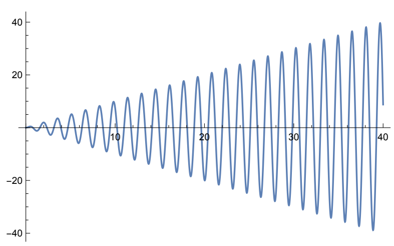

(d)