Math 2214, Fall 2014, Form A

1. If $x' = x^3$, and $x(1) = 1$, then $x(1.25)$ is

   (a) $e^{1/4}$.

   (b) $\sqrt{6}$.

   (c) $\sqrt{2}$.

   (d) $4^{1/3}$.

2. The solution of the initial value problem

$$(t-3)(t-5)y'' + 3y' = \tan t, \qquad y(2) = 0, \qquad y'(2) = 2,$$

   is guaranteed to exist on the interval

   (a) $(\pi/2, 3\pi/2)$.

   (b) $(3, 5)$.

   (c) $(\pi/2, 3)$.

   (d) $(-\pi/2, \pi/2)$.

3. The general solution of the system $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix},$$

   is

   (a) $c_1 e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

   (b) $c_1 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 t e^{2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

   (c) $c_1 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

   (d) $c_1 e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 (t e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{4t} \begin{pmatrix} 1 \\ -1 \end{pmatrix})$.

4. A nonlinear system is given by

$$x_1' = x_1^2 x_2 - x_1.$$

$$x_2' = x_2 x_1 - x_2^2.$$

The number of equilibrium points for this system is

(a) Three.

(b) One.

(c) Two.

(d) Four.

5. For the system

$$x' = x^2 - x + y,$$
$$y' = x^2 - y,$$

the point at $(1, 0)$ is a

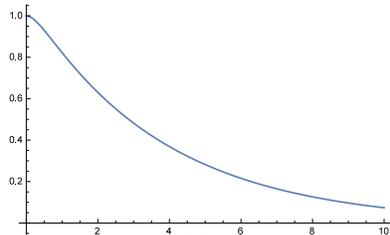(a) saddle.

(b) focus.

(c) unstable node.

(d) stable node.

6. Which of the following are linear equations?

(1) $y'' + \sin y = 1$.

(2) $y''' - y'' + e^t y = 0$.

(3) $y'/y = t^2$.

(4) $t^5 y'' + t^4 = \cos t$.
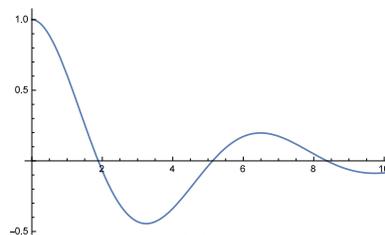
(a) All of the above.

(b) (2) only.

(c) (2) and (4).

(d) (2),(3), and (4).

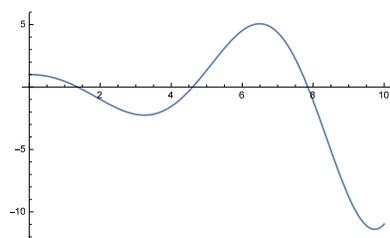7. Which of the following plots shows the solution of the problem

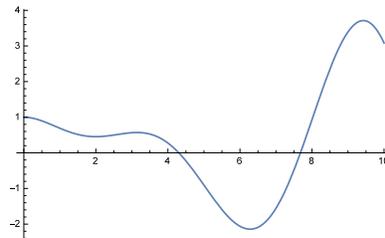$$y'' + 4y' + y = 0, \qquad y(0) = 1, \qquad y'(0) = 0?$$

(a)

(b)

(c)

(d)

8. An aquarium containing 20 gallons of water is connected to a pump which drives water through a filter and then pumps it back into the aquarium. The filter removes 90% of the pollutants passing through it. Water in the aquarium is pumped out at a rate of 0.2 gallons per minute through the filter and returned to the aquarium at the same rate. Initially, the aquarium contains 10 grams of pollutants, and the fish in the aquarium produce 0.5 grams of pollutants per minute. With the amount of pollutants $Q(t)$ measured in grams and the time t measured in minutes, the initial value problem for $Q(t)$ describing this situation is

   (a) $Q' = 0.5 - 0.009Q$, $Q(0) = 10$.

   (b) $Q' = 0.5 - 0.9Q$, $Q(0) = 10$.

   (c) $Q' = -0.18Q$, $Q(0) = 10 + 0.5t$.

   (d) $Q' = 0.5t - 0.18Q$, $Q(0) = 10$.

9. A particular solution of the equation $y'' + y = e^t + \sin t$ is given by

   (a) $te^t + t\sin(t)$.

   (b) $e^t/2 - t\cos(t)/2$.

   (c) $e^t/2 + \sin(t)/2$.

   (d) $te^t/2 + \cos(t)$.

10. Which of the following is not a solution of the equation $y''' - y = 0$?

   (a) $e^{-t/2}\cos(\sqrt{3}t/2)$.

   (b) e^{t-1}.

   (c) te^t.

   (d) $2e^t + e^{t+2}$.

11. You solve the initial value problem $y' = y + t^2$, $y(0) = 1$ using the Euler method with $h = 0.1$. Then the approximation you find for $y(0.2)$ is

    (a) 1.214.
    (b) 1.211.
    (c) 1.1.
    (d) 1.12.

12. A linear system is given by

$$x' = -x + by,$$

$$y' = x - 2y.$$

For the system to be asymptotically stable, the constant b must satisfy the following condition:

    (a) $b < -1/4$.
    (b) $b < 1/2$.
    (c) $b < 2$.
    (d) $b < 0$.