42nd Annual Virgina Tech Regional Math Contest

From 9:00am – 11:30am

October 22, 2022

1. Give all possible representations of 2022 as a sum of at least two consecutive positive integers and prove that these are the only representations.

2. Let A and B be the two foci of an ellipse and let P be a point on this ellipse. Prove that the focal radii of P (that is, the segments $\overline{AP}$ and $\overline{BP}$) form equal angles with the tangent to the ellipse at P.

3. Find all positive integers a, b, c, d, and n satisfying $n^a + n^b + n^c = n^d$ and prove that these are the only such solutions.

4. Calculate the exact value of the series $\displaystyle\sum_{n=2}^{\infty} \log(n^3+1) - \log(n^3-1)$ and provide justification.

5. Let A be an invertible $n \times n$ matrix with complex entries. Suppose that for each positive integer m, there exists a positive integer k_m and an $n \times n$ invertible matrix B_m such that $A^{k_m m} = B_m A B_m^{-1}$. Show that all eigenvalues of A are equal to 1.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a function whose second derivative is continuous. Suppose that f and f'' are bounded. Show that f' is also bounded.