HyBR: A Method for Solving Ill-Posed Inverse Problems in Image Processing

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MOTIVATION

Good image processing techniques are required in a variety of scientific applications such as biomedicine, astronomy and surveillance.

Basic Problem:
Given a corrupted, blurred image and some information about the image acquisition process, recover a high-resolution, detailed image.

The Main Challenge:
The problem is ill-posed, meaning small noise in the observed data may lead to significant errors in the computed solutions.

1 Introduction

Many image processing problems are mathematically modelled as:

$$\mathbf{b} = A\mathbf{x} + \mathbf{e}$$

where

$\mathbf{b} \in \Re^m$ represents the observed image

$\mathbf{x} \in \Re^n$ represents the true image

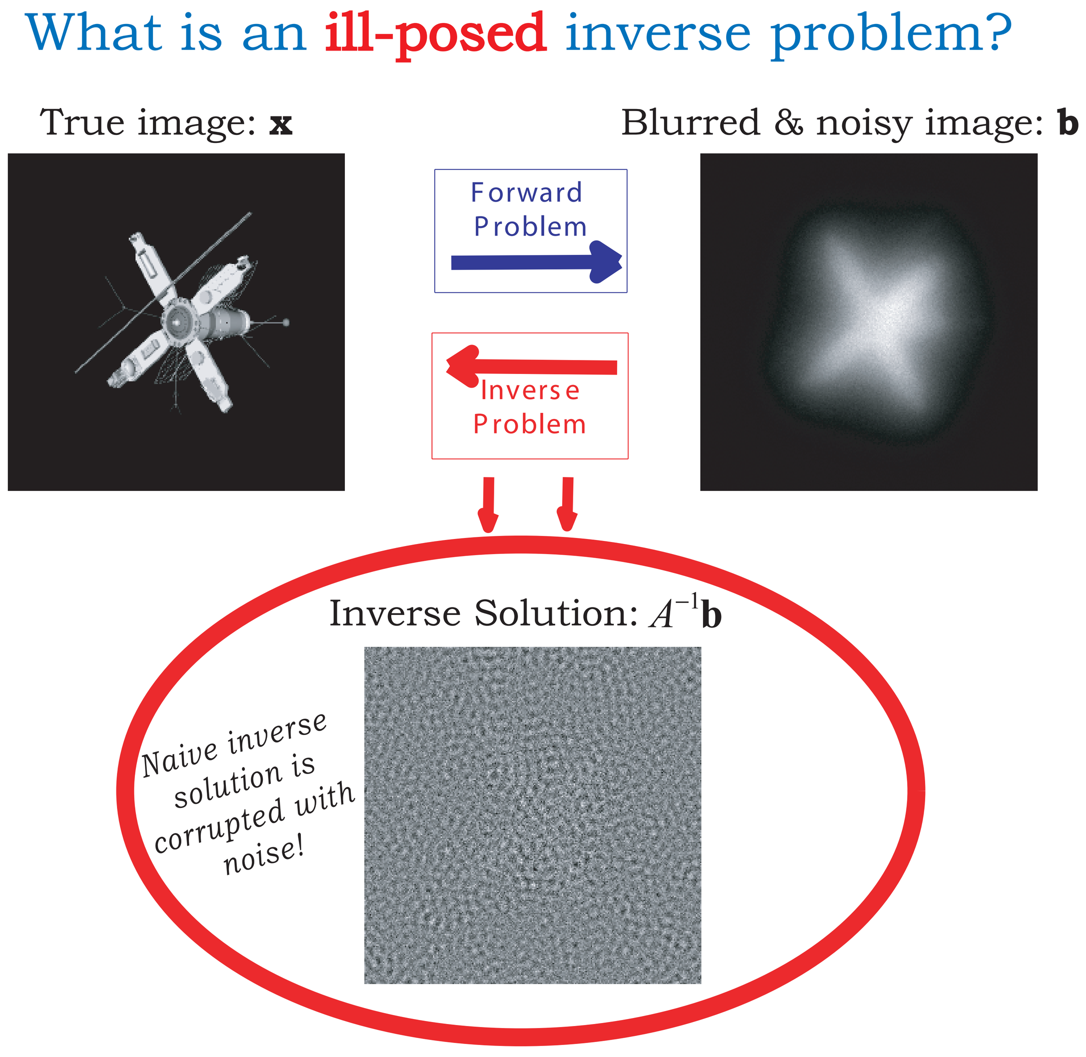
$A \in \Re^{m \times n}$ models the blurring process

$\mathbf{e} \in \Re^m$ represents noise in the data

What is an inverse problem?

The opposite of a forward problem!
That is, given $\mathbf{b}$ and A, compute $\mathbf{x}$.

What is an ill-posed inverse problem?



Remark: Matrix A is ill-conditioned.

Singular values decay to and cluster at 0, causing amplification of noise in the solution.

2 What is Regularization?

Goal: Modify the inversion process to avoid noise amplification

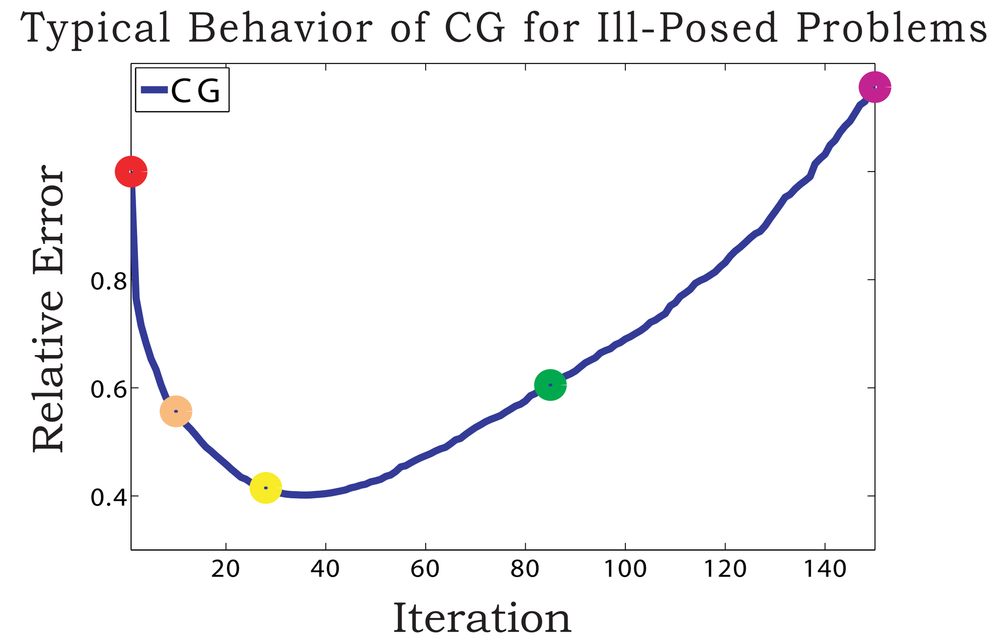
Two examples:

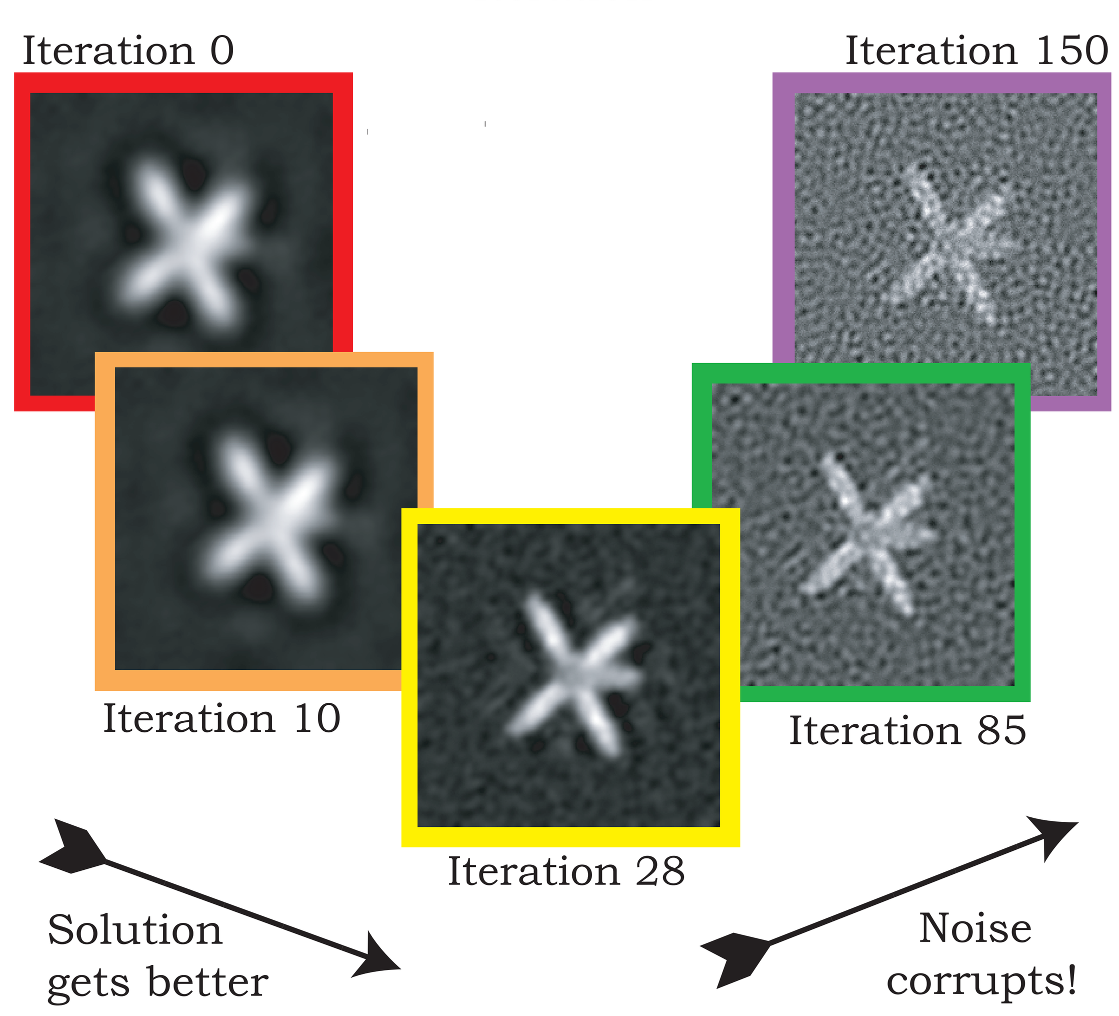
- Tikhonov Regularization

$$\min_{\mathbf{x}} \left\| \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} A \\ \lambda I \end{bmatrix} \mathbf{x} \right\|_2 \qquad \lambda \text{ - regularization parameter}$$

- Conjugate Gradient (CG)

$$\min_{\mathbf{x}} \|\mathbf{b} - A\mathbf{x}\|_2$$





3 HyBR Method

For $k = 1, 2, \ldots$ compute

1. Lanczos Bidiagonalization

$$U = [\mathbf{u}_1 \;\; \mathbf{u}_2 \;\; \cdots \;\; \mathbf{u}_k \;\; \mathbf{u}_{k+1}], \quad \mathbf{u}_1 = \mathbf{b} / \|\mathbf{b}\|$$

$$V = [\mathbf{v}_1 \;\; \mathbf{v}_2 \;\; \cdots \;\; \mathbf{v}_k]$$

$$B = \begin{bmatrix} \alpha_1 & & & \\ \beta_2 & \alpha_2 & & \\ & \ddots & \ddots & \\ & & \beta_k & \alpha_k \\ & & & \beta_{k+1} \end{bmatrix}$$

where U and V have orthonormal columns, and

$$AV = UB$$

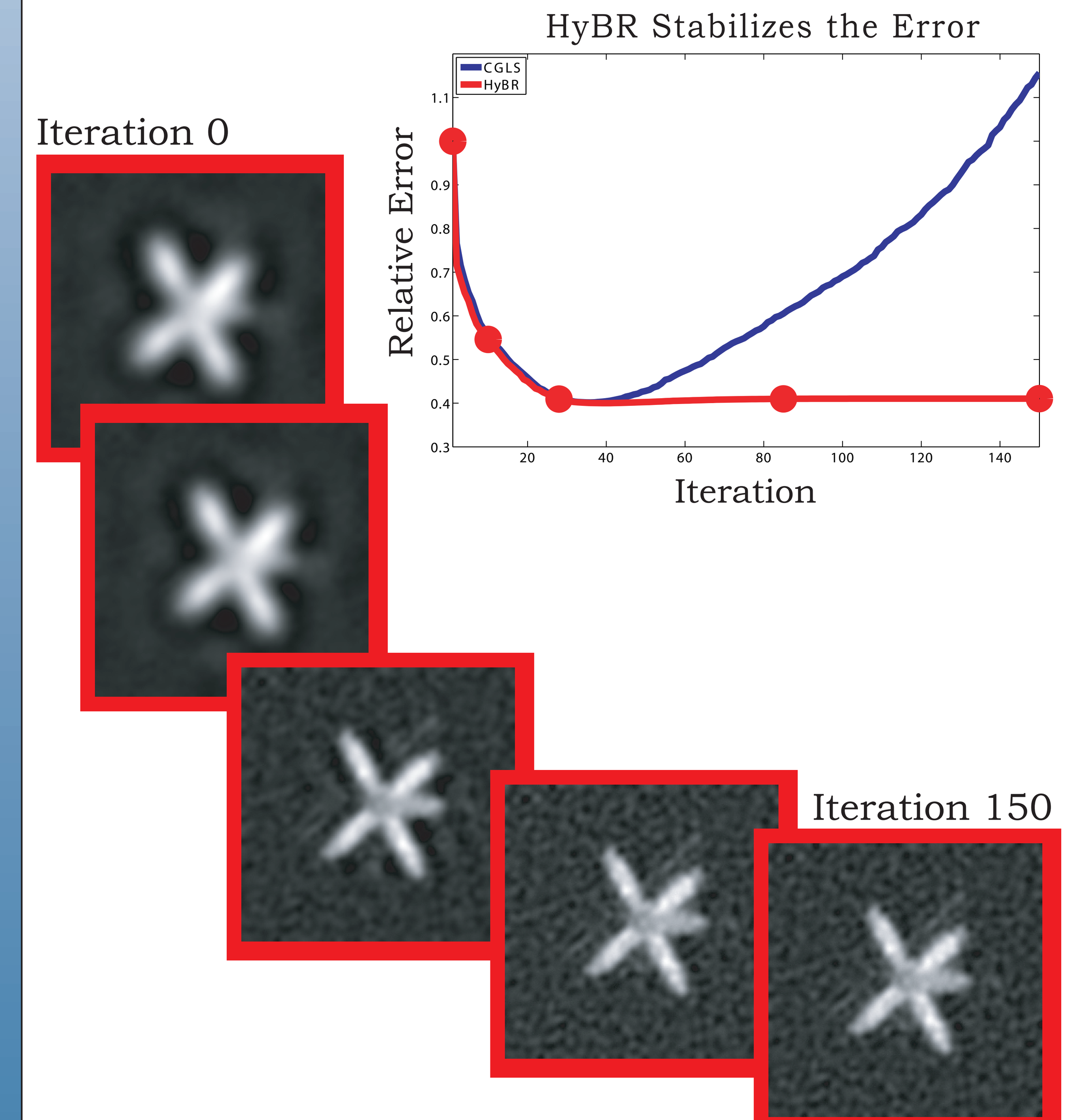
2. Solve the projected LS problem with Tikhonov regularization

$$\min_{\mathbf{x} \in R(V)} \|\mathbf{b} - A\mathbf{x}\|_2 = \min_{\mathbf{f}} \|U^T\mathbf{b} - B\mathbf{f}\|_2$$

where $\mathbf{x} = V\mathbf{f}$

Remarks:

* A ill-conditioned ➡ B ill-conditioned
* B is much smaller than A
* Standard techniques (e.g. GCV) to find λ and a stopping point
* GCV tends to over-smooth solutions so use weighted GCV



4 Conclusions

* HyBR stabilizes noise
* Stopping criteria not as crucial
* Automated HyBR computes fairly good results for any ill-posed problem, with little user input
* Future direction includes incorporating non-negativity constraints

References

J. Chung, J. Nagy, D. O'Leary. ETNA, (2007).

M. E. Kilmer, D. O'Leary. SIAM J. Matrix Anal. Appl., (2001).