Honghu Liu

Contact Information

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Education

Ph.D. in Mathematics, Indiana University Bloomington, 2013.
Advisor: Prof. Shouhong Wang
Dissertation: On Some Dynamic Transition Problems

B.S. in Mathematics, Sichuan University, Sichuan, China, 2007.

Employment

8/2022–present	Associate Professor, Virginia Tech.
8/2015–8/2022	Assistant Professor, Virginia Tech.
3/2013–7/2015	Postdoctoral scholar, Department of Atmospheric & Oceanic Sciences, UCLA. Mentors: Prof. Michael Ghil and Dr. Mickaël D. Chekroun.

Research Interests

- Low-dimensional closures for nonlinear deterministic/stochastic PDEs
- Stochastic parameterizations
- Stochastic invariant manifolds and their approximations
- Bifurcation analysis, phase transition and pattern formation
- Geophysical fluid dynamics
- Data-driven modeling
- Optimal control of nonlinear evolution equations
- Delay differential equations

Grants

Lead PI, National Science Foundation Grant: DMS-1616450, $214,483, 5/17/16–7/31/20, *Collaborative research: Non-Markovian reduction of nonlinear stochastic partial differential equations, and applications to climate dynamics*. This is a collaborative proposal with Dr. M. D. Chekroun at UCLA. The lead organization is Virginia Tech. The amount listed above is the Virginia Tech portion of the budget.

Solo PI, National Science Foundation Grant: DMS-2108856, $113,763, 6/1/21–5/31/24, *Parameterization and reduction for nonlinear stochastic systems with applications to fluid dynamics*.

Lead PI, Virginia Tech College of Science Dean's Discovery Fund, $22,265.52, 5/15/17–6/30/18, *Stochastic nonlinear reduced order modeling of the El Niño Southern Oscillation (ENSO)*. Co-PI: Prof. T. Iliescu.

Publications

Research Monographs

M2. M. D. Chekroun, H. Liu, and S. Wang, *Stochastic Parameterizing Manifolds and Non-Markovian Reduced Equations: Stochastic Manifolds for Nonlinear SPDEs II*. Springer Briefs in Mathematics, Springer, New York, xvii+129 pp., 2015.

M1. M. D. Chekroun, H. Liu, and S. Wang, *Approximation of Stochastic Invariant Manifolds: Stochastic Manifolds for Nonlinear SPDEs I*. Springer Briefs in Mathematics, Springer, New York, xv+127 pp., 2015.

Journal Articles ($*$ indicates a student author)

J19. M. D. Chekroun, H. Liu, J. C. McWilliams, and S. Wang, Transitions in stochastic non-equilibrium systems: Efficient reduction and analysis. *Journal of Differential Equations*, 346, 145-204, 2023. DOI: 10.1016/j.jde.2022.11.025.

J18. M. Chung, J. Krueger, and H. Liu, Least-squares finite element method for ordinary differential equations. *Journal of Computational and Applied Mathematics*, 418, 114660, 2023. DOI: 10.1016/j.cam.2022.114660. arXiv preprint: `https://arxiv.org/abs/2109.15133`

J17. M. D. Chekroun, I. Koren, H. Liu, and Huan Liu. Generic generation of noise-driven chaos in stochastic time delay systems: Bridging the gap with high-end simulations. *Science Advances*, 8, eabq7137, 19 pp., 2022. DOI: 10.1126/sciadv.abq7137.

J16. B. Koc*, C. Mou*, H. Liu, Z. Wang, G. Rozza, and T. Iliescu, Verifiability of the data-driven variational multiscale reduced order model. *Journal of Scientific Computing*, 93, Article number: 54, 26 pp., 2022. DOI: 10.1007/s10915-022-02019-y.

J15. N. Chen, Y. Li* and H. Liu, Conditional Gaussian nonlinear system: a fast preconditioner and a cheap surrogate model for complex nonlinear systems. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 32, 053122, 30 pp., 2022. DOI: 10.1063/5.0081668 (This paper was selected as an **Editor's Pick**)

J14. N. Chen, H. Liu, F. Lu, Shock trace prediction by reduced models for viscous stochastic Burgers equations. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 32, 043109, 17 pp., 2022. DOI: 10.1063/5.0084955

J13. M. D. Chekroun, H. Liu, and J. C. McWilliams, Stochastic rectification of fast oscillations on slow manifold closures. *Proceedings of the National Academy of Sciences*, 118, e2113650118, 2021.

J12. M. D. Chekroun, I. Koren, and H. Liu, Efficient reduction for diagnosing Hopf bifurcation in delay differential systems: Applications to cloud-rain models. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 30, 053130, 27 pp., 2020. DOI: 10.1063/5.0004697

J11. C. Mou*, H. Liu, D. R. Wells, and T. Iliescu, Data-driven correction reduced order models for the quasi-geostrophic equations: A numerical investigation. *International Journal of Computational Fluid Dynamics*, 34, 147–159, 2020. DOI: 10.1080/10618562.2020.1723556

J10. M. D. Chekroun, H. Liu, and J. C. McWilliams, Variational approach to closure of nonlinear dynamical systems: Autonomous case. *Journal of Statistical Physics*, 179, 1073–1160, 88 pp., 2020. DOI: 10.1007/s10955-019-02458-2

J9. T. Iliescu, H. Liu, and X. Xie*, Regularized reduced order models for a stochastic Burgers equation, *International Journal of Numerical Analysis & Modeling*, 15, 594–607, 2018.

J8. N. Boers, M. D. Chekroun, H. Liu, D. Kondrashov, D.-D. Rousseau, A. Svensson, M. Bigler, and M. Ghil, Inverse stochastic-dynamic models for high resolution Greenland ice core records, *Earth System Dynamics*, 8, 1171–1190, 2017. DOI: 10.5194/esd-8-1171-2017

J7. M. D. Chekroun, A. Kröner, H. Liu, Galerkin approximations of nonlinear optimal control problems in Hilbert spaces. *Electronic Journal of Differential Equations*. Vol. 2017, No. 189, 1-40, 2017.

J6. M. D. Chekroun, H. Liu, and J. C. McWilliams, The emergence of fast oscillations in a reduced Primitive Equation model and its implications for closure theories. *Computers and Fluids*, 151, 3–22, 2017. DOI: 10.1016/j.compfluid.2016.07.005

J5. M. D. Chekroun, M. Ghil, H. Liu, and S. Wang, Low-dimensional Galerkin approximations of nonlinear delay differential equations. *Disc. Cont. Dyn. Sys. A*, 36, 4133–4177, 2016. DOI: 10.3934/dcds.2016.36.4133

J4. M. D. Chekroun and H. Liu, Finite-horizon parameterizing manifolds, and applications to suboptimal control of nonlinear parabolic PDEs. *Acta Appl. Math.*, 135, 81–144, 2015.

J3. H. Liu, T. Sengul, S. Wang, and P. Zhang, Dynamic transitions and pattern formations for Cahn-Hilliard model with long-range repulsive interactions. *Comm. Math. Sci.*, 13, 1289–1315, 2015. DOI: 10.4310/CMS.2015.v13.n5.a10

J2. H. Liu, T. Sengul, and S. Wang, Dynamic transitions for quasilinear systems and Cahn-Hilliard equation with Onsager mobility. *J. Math. Phys.*, **53**:023518, 31 pp., 2012. DOI: 10.1063/1.3687414

J1. H. Liu, Phase transitions of a phase field model. *Disc. Cont. Dyn. Sys. B*, 16, 883–894, 2011. DOI: 10.3934/dcdsb.2011.16.883

Refereed Book Chapters

BC3. W. Snyder*, C. Mou, H. Liu, O .San, R. De Vita, and T. Iliescu, Reduced Order Model Closures: A Brief Tutorial. In F. Carapau, A. Vaidya, editors, *Recent Advances in Mechanics and Fluid-Structure Interaction with Applications*, pages 167–193. Birkhäuser, 2022. DOI: 10.1007/978-3-031-14324-3

BC2. M. D. Chekroun and H. Liu, Optimal management of harvested population at the edge of extinction. In J. Kotas, editor, *Advances in Nonlinear Biological Systems: Modeling and Optimal Control*, chapter 2, pages 35–72. American Institute of Mathematical Sciences, 2020.

BC1. M. D. Chekroun, A. Kröner, and H. Liu, Galerkin approximations for the optimal control of nonlinear delay differential equations. In D. Kalise, K. Kunisch, Z. Rao (Eds.), *Hamilton-Jacobi-Bellman Equations: Numerical Methods and Applications in Optimal Control*, chapter 4, pages 61–96. De Gruyter, Berlin, Boston, 2018. DOI: 10.1515/9783110543599-004

Refereed Conference Proceedings

C1. M. D. Chekroun and H. Liu, Post-processing finite-horizon parameterizing manifolds for optimal control of nonlinear parabolic PDEs. *the Proceedings of 55th IEEE Conference on Decision and Control*, 1411–1416, 2016. DOI: 10.1109/CDC.2016.7798464

Work in Progress

J20. M. D. Chekroun, H. Liu, and J. C. McWilliams, Variational Markovian and non-Markovian closures for stochastic partial differential equations. *In preparation.* (38 pp.)

Teaching Experience

Fall 2022	Fourier Series & PDEs, Math-4425	

Spring 2022	Topic Course on Stochastic Dynamics, Math-5414 Intro. Multivariable Calculus, Math-2204
Fall 2021	Intro. Multivariable Calculus (honors section), Math-2204H
Spring 2021	Intro. Multivariable Calculus, Math-2204
Fall 2020	Intro. Multivariable Calculus, Math-2204, Two sessions
Spring 2020	No teaching assignment
Fall 2019	Intro. Multivariable Calculus, Math-2204, Two sessions
Spring 2019	Differential equations, Math-5246 Applied Mathematical Modeling, Math-4454
Fall 2018	Intro. Multivariable Calculus, Math-2204
Spring 2018	Applied PDE sequence, Math-5426
Fall 2017	Applied PDE sequence, Math-5425
Spring 2017	Applied PDE sequence, Math-5426
Fall 2016	Applied PDE sequence, Math-5425
Spring 2016	Intro. Multivariable Calculus, Math-2204
Fall 2015	Intro. Multivariable Calculus, Math-2204

Advising

- Trevor Norton, Master's thesis, 2018.
- Xuping Xie, graduate research project which led to the publication [J9], 2017.
- Changhong Mou, graduate research project which led to the publication [J11], 2019.
- Birgul Koc & Changhong Mou, graduate research project which led to the submitted manuscript [J15], 2020.
- Qixuan Xing, undergraduate research, Summer 2019 and Spring 2020.
- Matthew Pinho, undergraduate research, Spring 2020.

Conferences and Mini-symposia Organized

7/6-8/2022	Conference on "*Accurate ROMs for Industrial Applications at Virginia Tech*" (Co-organized with Traian Iliescu, John Burns, Raffaella De Vita, Angelo Iollo)
4/11-15/2022	Minisymposia on *Information Theory, Data Assimilation and Stochastic Models for Multiscale Systems Part I-III* at 2022 SIAM Conference on Uncertainty Quantification (Co-organized with Nan Chen at U. Wisconsin-Madison)
2019 Fall & 2020 Spring	Served as the coordinator of the *Fluids Seminars* that runs weekly in the Mathematics Department at Virginia Tech.
9/14/2019-9/15/2019	Special Session on *"Classical and Geophysical Fluid Dynamics: Modeling, Reduction and Simulation"* at the AMS Fall Central Sectional Meeting, University of Wisconsin-Madison, Madison, WI. (Co-organized with Nan Chen).

7/5/2018-7/9/2018	Special Session on *"Classical and Geophysical Fluid Dynamics: Modeling, Analysis and Reduction"* at the 12th AIMS Conference on Dynamical Systems, Differential Equations and Applications, Taipei, Taiwan. (Co-organized with Mickaël Chekroun, Taylan Sengul, and Shouhong Wang).
6/26/2017-6/28/2017	Conference on *"Classical and Geophysical Fluid Dynamics: Modeling, Reduction and Simulation"*, Virginia Tech (Co-organized with Jeff Borggaard, Mickaël Chekroun, Traian Iliescu, Shouhong Wang and Lizette Zietsman; I served as the chair of the organization commitee).
2/27/2017-3/3/2017	Minisymposia on *"Reduced Order Models for Fluids: Achievements and Open Problems"* at 2017 SIAM Conference on Computational Science and Engineering (Co-organized with Jeff Borggaard, Traian Iliescu, and Lizette Zietsman).
7/2-3/2016	Special Session on *"Stochastic Modeling in Fluid Dynamics: Theory and Approximation"* at the 11th AIMS Conference on Dynamical Systems, Differential Equations and Applications (Co-organized with Roger Temam and Chuntian Wang).
12/7-10/2015	Minisymposium on *"Deterministic and Stochastic Aspects of Fluid Dynamics"* at SIAM Conference on Analysis of Partial Differential Equations (Co-organized with Michele Coti Zelati, Roger Temam and Chuntian Wang).

Seminars and Invited Talks

3/26-27/2022	Special session on Modeling and Forecasting Complex Turbulent Systems, AMS Spring Central Sectional Meeting, Virtual.
3/11-13/2022	Special session on Advances in Difference, Differential, Fractional Differential and Dynamic Equations with Applications, AMS Spring Southeastern Sectional Meeting, University of Virginia, Charlottesville, VA. (The whole conference was cancelled due to Covid-19)
12/13-17/2021	Session NG12A: Advances in Computational Analysis in Geophysical Processes: Applied Math Perspectives on Multiscale and Stochastic Models I, American Geophysical Union Fall Meeting. (The talk was delivered virtually)
11/24/2021	Dynamical System Seminar talk, Department of Mathematics, China University of Mining and Technology. (The seminar was delivered virtually)
9/18-19/2021	Minisymposia on Reduced Order Modeling in the Age of Data, SIAM Southeastern Atlantic Section Conference. (The talk was delivered virtually)
4/9/2021	Comp Math Seminar talk, Department of Mathematical and Statistical Sciences, Clemson University. (The seminar was delivered virtually)
9/2/2020	BEAM seminar talk, College of Engineering, Virginia Tech. (The seminar was delivered virtually)

8/3-14/2020	Special session on Mathematical Modeling of the Paleoclimate - Tales for the Future Climate, SIAM Conference on Mathematics of Planet Earth. (The conference was held virtually)
6/5-9/2020	Special session on Advances in Mathematical Physics, The 13th AIMS Conference on Dynamical Systems, Differential Equations and Applications, Atlanta, GA. (The whole conference was rescheduled to June 2022 due to Covid-19)
6/5-9/2020	Special session on Stochastic Modeling in Biological, Physical and Social Sciences: Theory and Applications, The 13th AIMS Conference on Dynamical Systems, Differential Equations and Applications, Atlanta, GA. (The whole conference was rescheduled to June 2022 due to Covid-19)
3/13-15/2020	Special session on Advances in High and Infinite Dimensional Stochastic Analysis, AMS Sectional Meeting, University of Virginia, Charlottesville, VA. (The whole conference was cancelled due to Covid-19)
10/7-11/2019	Workshop on Nonlinear and Stochastic Methods in Climate and Geophysical Fluid Dynamics, Institut Henri Poincaré (IHP), Paris, France.
9/14-15/2019	Special session on Classical and Geophysical Fluid Dynamics: Modeling, Reduction and Simulation, AMS Fall Central Sectional Meeting, University of Wisconsin-Madison, Madison, WI.
6/2-5/2019	International Conference on Recent Advances in Fluid Dynamics and Nonlinear Dynamics, Sichuan University, Chengdu, China.
2/25-3/1/2019	Minisymposia on Reduced Order Models for Fluids: Achievements and Open Problems, SIAM Conference on Computational Science and Engineering, Spokane Convention Center, Spokane, WA.
2/13/2019	Fluids Seminar, Department of Mathematics, Virginia Tech, Blacksburg, VA.
12/7/2018	Colloquium, Department of Mathematics, Old Dominion University, Norfolk, Virginia.
9/29-30/2018	Special session on Recent Analytic and Numeric Results on Nonlinear Evolution Equations, AMS Fall Eastern Sectional Meeting, University of Delaware, Newark, DE.
9/12-14/2018	International Union of Theoretical and Applied Mechanics Symposium: Stochastic approaches to understanding transitions in Fluid Flows, Cornell University, Ithaca, New York.
7/5-9/2018	Special session on Stochastic Modeling in Biology, Phase Transitions and Fluid Dynamics: Theory and Approximation, 12th AIMS Conference on Dynamical Systems, Differential Equations and Applications, Taipei, Taiwan.
11/20-24/2017	Workshop on Nonlinear and Stochastic Problems in Atmospheric and Oceanic Prediction, Banff International Research Station for Mathematical Innovation and Discovery (BIRS), Alberta, Canada. (My trip was cancelled due to family reasons)
5/22-25/2017	Minisymposium on Optimization with PDEs: Theory and Numerics, 2017 SIAM Conference on Optimization, Vancouver, Canada.

12/12/2016	55th IEEE Conference on Decision and Control, Las Vegas, NV.
07/05/2016	Special Session on Nonlinearity in Climate and the Geosciences, A Special Session Honoring Peter D. Lax, 11^{th} AIMS Conference on Dynamical Systems, Differential Equations and Applications, Orlando, FL.
07/02/2016	Special Session on Stochastic Modeling in Fluid Dynamics: Theory and Approximation, 11^{th} AIMS Conference on Dynamical Systems, Differential Equations and Applications, Orlando, FL.
03/09/2016	Theoretical and Mathematical Physics Seminar, Department of Mathematics, Indiana University, Bloomington, IN.
11/11/2015	Applied Analysis Seminar, Department of Mathematics, Virginia Tech, Blacksburg, VA.
1/20/2015	Colloquium, Department of Mathematics, Virginia Tech, Blacksburg, VA.
12/15-19/2014	Special Session on Stochastic Modeling and Complex System Approaches to Nonlinear Geophysical Systems, American Geophysical Union Fall Meeting, San Francisco, CA.
11/21/2014	Colloquium, Department of Mathematics, Virginia Tech, Blacksburg, VA.
4/4-6/2014	Special Session on Stochastics and PDEs, AMS 2014 Western Spring Sectional Meeting, Albuquerque, NM.
10/5/2013	Special Session on Partial Differential Equations from Fluid Mechanics, AMS 2013 Fall Southeastern Sectional Meeting, Louisville, KY.
11/16/2012	CCAM Lunch Seminars, Center for Computational & Applied Mathematics, Purdue University, West Lafayette.
11/12/2012	PDE seminar, Department of Mathematics, Indiana University, Bloomington, IN.
7/1/2012	Special Session on Advances in Classical and Geophysical Fluid Dynamics, 9^{th} AIMS International Conference, Orlando, Florida.
4/1/2011	Graduate student seminar, Department of Mathematics, Indiana University, Bloomington, IN.
3/7/2011	PDE seminar, Department of Mathematics, Indiana University, Bloomington, IN.
11/5-7/2010	Special Session on Deterministic and Stochastic Partial Differential Equations, AMS 2010 Fall Central Section Meeting, Notre Dame, IN.

Peer Review Service

- I am a reviewer for AMS's Mathematical Reviews
- I have served as an external reviewer for a research proposal submitted to the Dutch Research Council, the Netherlands, 2021.
- I served as a proposal panelist for NSF Applied Mathematics program, Spring 2022.

I have also served as a peer reviewer for the following journals:

- Applied Mathematics and Computation
- Applied Numerical Mathematics
- Communications in Mathematical Sciences
- Discrete and Continuous Dynamical Systems - Series B
- Dynamical Systems
- Entropy
- International Journal of Bifurcation and Chaos
- International Journal of Non-Linear Mechanics
- International Journal of Numerical Analysis & Modeling
- Journal of Applied Mathematics and Physics (ZAMP)
- Journal of Computational and Applied Mathematics
- Journal of Mathematical Analysis and Applications
- Journal of Nonlinear Mathematical Physics
- Modelling and Simulation in Materials Science and Engineering
- Physica D: Nonlinear Phenomena
- Proceedings of the Royal Society Proceedings A
- Research in the Mathematical Sciences
- Transactions of the Canadian Society for Mechanical Engineering

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Committee Service

- Differential Equations Faculty Position search committee chair, 2022 Fall
- Stochastic Differential Equations and Analysis Faculty Position search committee, 2021 Fall
- Stochastic Analysis Faculty Position search committee, 2016 Fall
- Postdoc search committee, 2021 Spring
- College of Science Dean's Discovery Fund review panel, 2021 Spring
- College of Science Dean's Discovery Fund review panel, 2020 Spring
- PhD Thesis Committee for Changhong Mou, 2021
- PhD Thesis Committee for Birgul Koc, 2021
- Master's Thesis Committee for Yichen Li, 2020
- Master's Thesis Committee for Changhong Mou, 2018