AN NET MAP PRESENTATION FOR $f(z) = \frac{3z^4-2z^2+3}{4(z^3-z)}$

We construct a finite subdivision rule for

$$f(z) = \frac{3z^4-2z^2+3}{4(z^3-z)} = z - \frac{z^4-2z^2-3}{4(z^3-z)} = z - \frac{p(z)}{p'(z)}.$$

We have that

$$f'(z) = \frac{p(z)p''(z)}{p'(z)^2} = \frac{(z^2-3)(z^2+1)4(3z^2-1)}{(4(z^3-z))^2}.$$

Fixed Points: $\pm\sqrt{3}$, $\pm i$, ∞
Critical Points: $\pm 1/\sqrt{3}$, $\pm\sqrt{3}$, $\pm i$
Postcritical Points: $\pm\sqrt{3}$, $\pm i$
$f(-1) = f(0) = f(1) = \infty$

We see that f maps the closed interval $[\sqrt{3}, \infty]$ homeomorphically to itself. Using the fact that the only real critical points of f have multiplicity 2 and are at $\pm 1/\sqrt{3}$ and $\pm\sqrt{3}$, we see that f maps $\mathbb{R} \cup \{\infty\}$ to $[-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty]$ in 4-to-1 fashion. Similarly, f maps $\mathbb{R}i \cup \{i\infty\}$ to $[-i\infty, -i] \cup [i, i\infty]$ in 2-to-1 fashion. In addition, there are two disjoint arcs in $\mathbb{C}$ which meet $\mathbb{R} \cup \mathbb{R}i$ at ± 1 which f maps homeomorphically to $[-i\infty, -i] \cup [i, i\infty]$.

We conclude that the union of the intervals $[-\infty, -\sqrt{3}]$, $[\sqrt{3}, \infty]$, $[-i\infty, -i]$ and $[i, i\infty]$ is a channel diagram Δ_0 for f. A portion of Δ_0 appears as a union of four black line segments in Figure 1, which was drawn using Mandel (www.mndynamics.com). Moreover, $f^{-1}(\Delta_0)$ is the union of $\mathbb{R}$, $\mathbb{R}i$, $\{\infty\}$ and the two arcs of the above paragraph. This proves that the finite subdivision rule presentation for f at the top of Figure 2 is correct up to homeomorphism.

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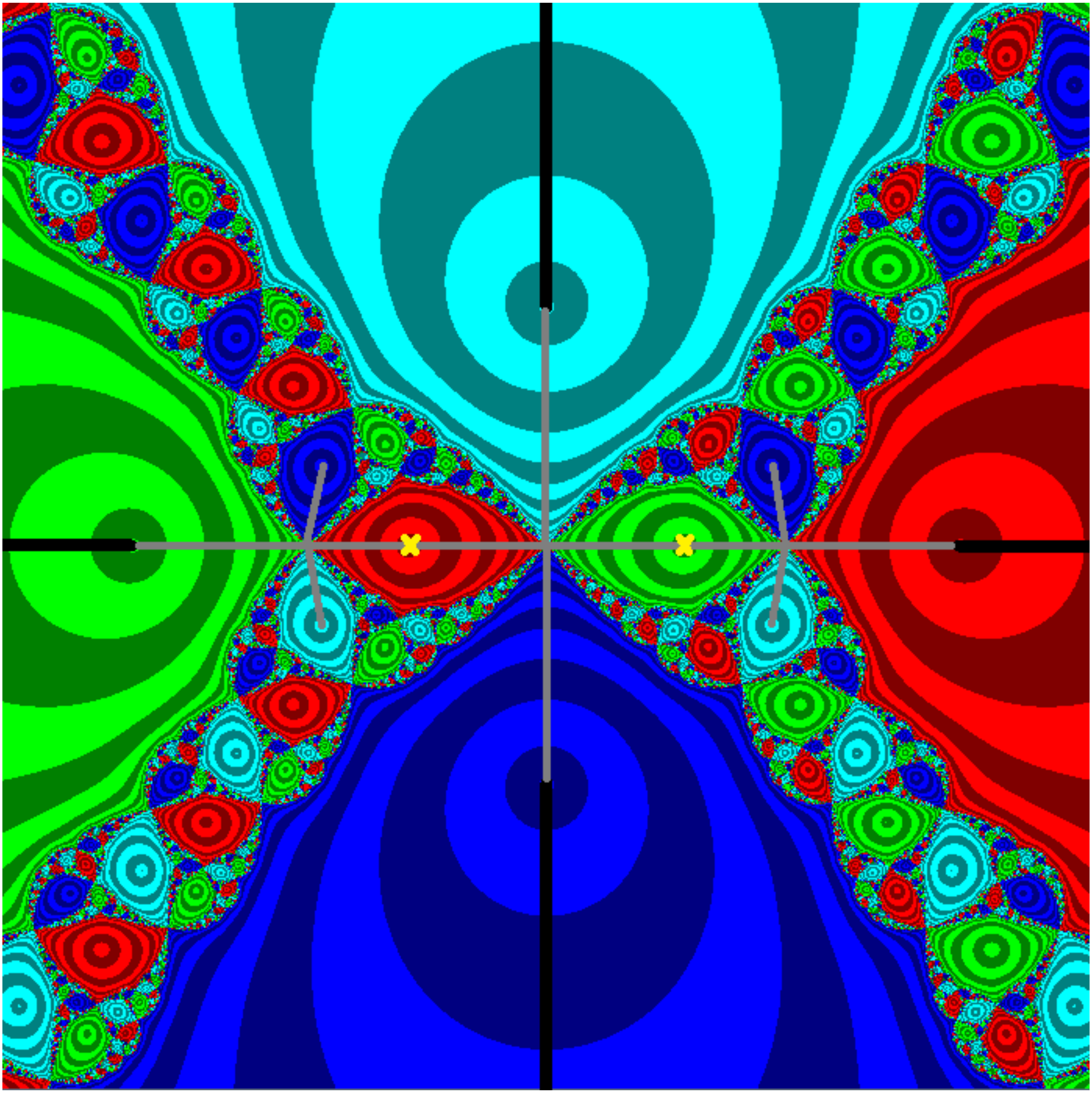
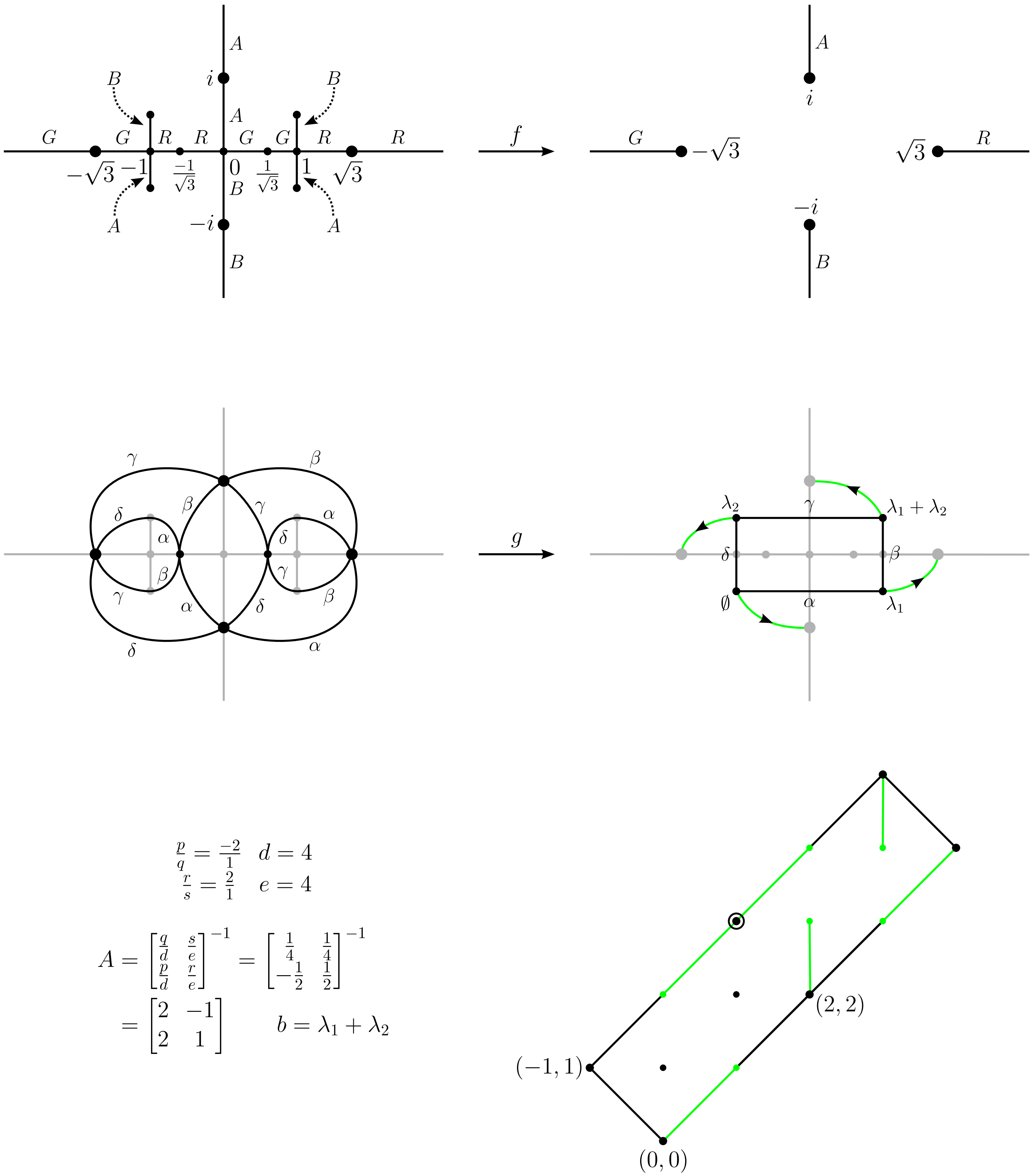


FIGURE 1. Constructing a finite subdivision rule for f



$$\frac{p}{q} = \frac{-2}{1} \quad d = 4$$
$$\frac{r}{s} = \frac{2}{1} \quad e = 4$$

$$A = \begin{bmatrix} \frac{q}{d} & \frac{s}{e} \\ \frac{p}{d} & \frac{r}{e} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}^{-1}$$
$$= \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \qquad b = \lambda_1 + \lambda_2$$

FIGURE 2. Constructing an NET map presentation for f