AN NET MAP PRESENTATION FOR $f(z) = \frac{2z^3+1}{3z^2}$

We construct a finite subdivision rule for
$$f(z) = \frac{2z^3+1}{3z^2} = z - \frac{z^3-1}{3z^2} = z - \frac{p(z)}{p'(z)}.$$
We have that
$$f'(z) = \frac{p(z)p''(z)}{p'(z)^2} = \frac{(z^3-1)6z}{(3z^2)^2}.$$
Fixed Points: 1, ω, ω^2, ∞ $\qquad \omega = (-1+\sqrt{-3})/2$
Critical Points: 0, 1, ω, ω^2
Postcritical Points: 1, ω, ω^2, ∞

We see that f maps the closed interval $[1, \infty]$ homeomorphically to itself. Using the fact that the only real critical points of f have multiplicity 2 and are at 0 and 1, we see that f maps the closed interval $[-\frac{1}{2}, \infty]$ to $[1, \infty]$ in 3-to-1 fashion. Because $f(\omega z) = \omega f(z)$, it follows that f maps the closed interval $[-\frac{1}{2}\omega^n, \omega^n\infty]$ to $[\omega^n, \omega^n\infty]$ in 3-to-1 fashion for $n \in \{0, 1, 2\}$.

We conclude that the union of the intervals $[\omega^n, \omega^n\infty]$ for $n \in \{0, 1, 2\}$ is a channel diagram Δ_0 for f. A portion of Δ_0 appears as a union of three black line segments in Figure 1, which was drawn using Mandel (www.mndynamics.com). Moreover, $f^{-1}(\Delta_0)$ is the union of the intervals $[-\frac{1}{2}\omega^n, \omega^n\infty]$ for $n \in \{0, 1, 2\}$. This proves that the finite subdivision rule presentation for f in Figure 2 is correct.

We note that this NET map is the reciprocal of the cubic map in Lodge's thesis.

Date: January 22, 2016.

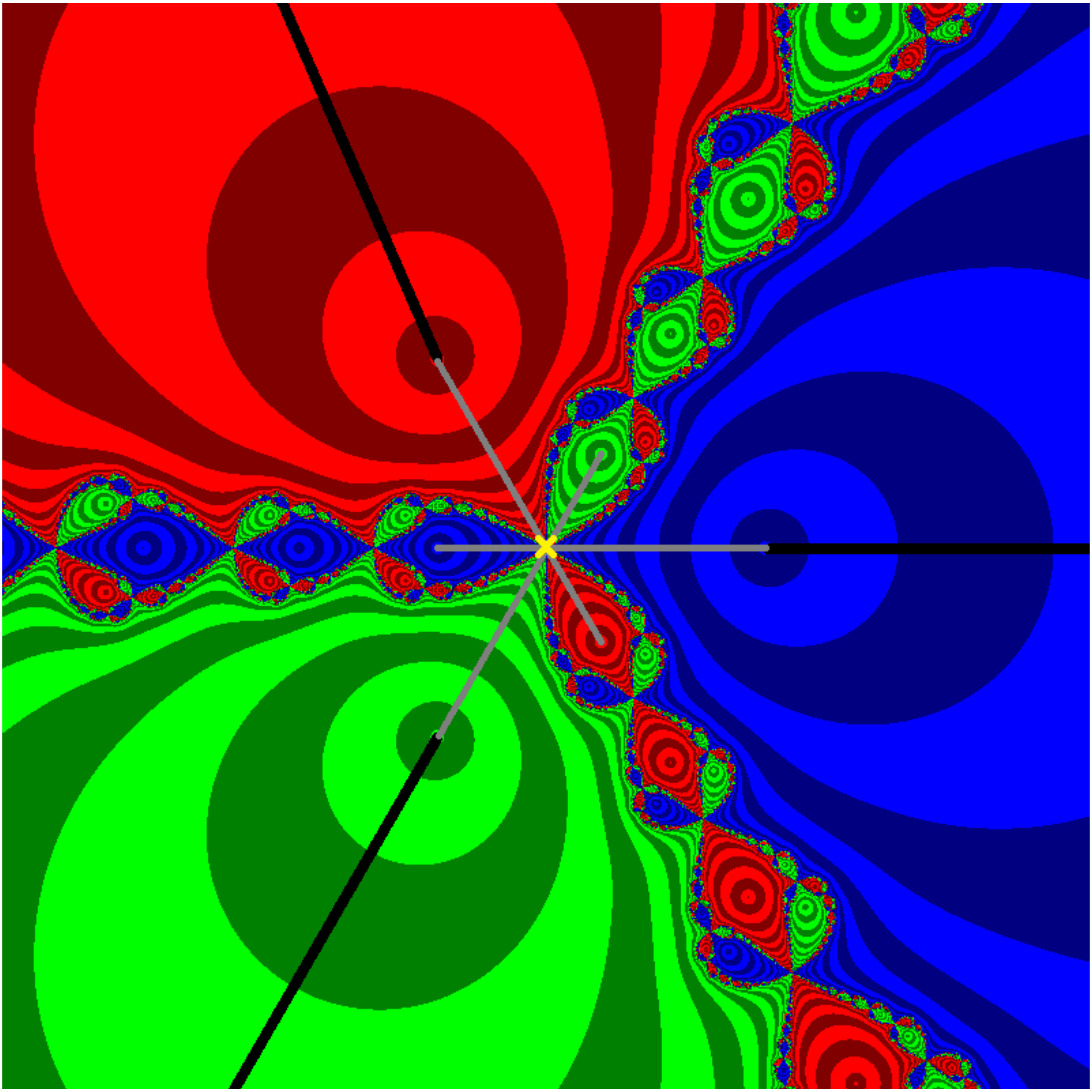
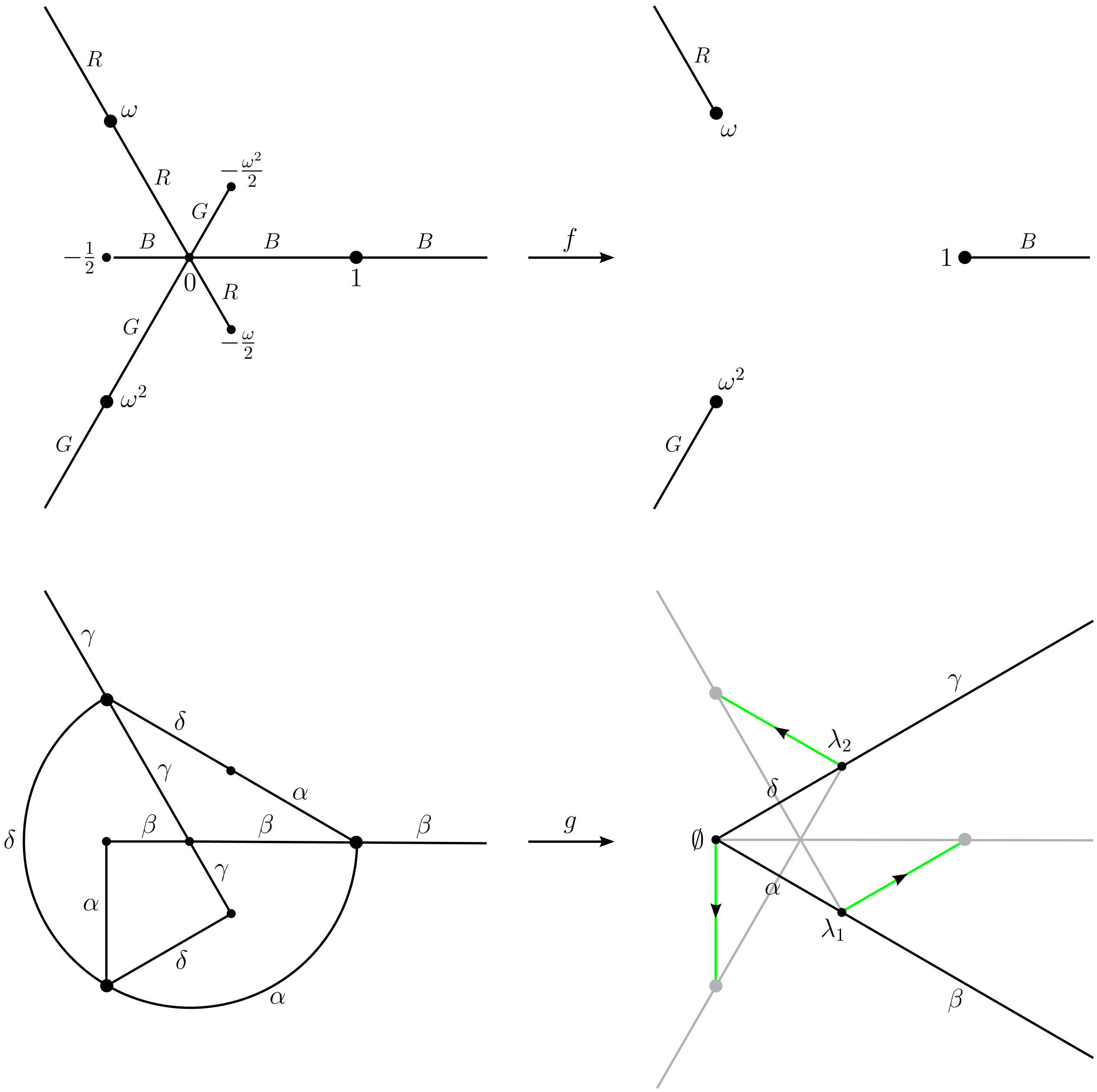


FIGURE 1. Constructing a finite subdivision rule for f



$$\tfrac{p}{q} = \tfrac{-1}{2} \quad d = 3$$
$$\tfrac{r}{s} = \tfrac{1}{1} \quad e = 3$$

$$A = \begin{bmatrix} \frac{q}{d} & \frac{s}{e} \\ \frac{p}{d} & \frac{r}{e} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}^{-1}$$
$$= \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} \qquad b = \lambda_1$$

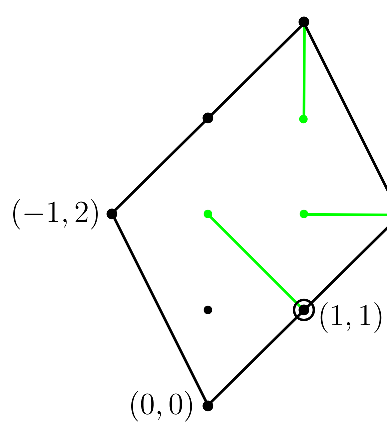


FIGURE 2. Constructing an NET map presentation for f