A NET MAP PRESENTATION FOR THE CUBIC NET MAP IN LODGE'S THESIS

We derive a presentation for the degree 3 NET map in Lodge's thesis:

$$f(z) = \frac{3z^2}{2z^3+1}.$$

Note that $f(\omega z) = \omega^2 f(z)$, where $\omega = e^{2\pi i/3}$. We compute:

$$f'(z) = \frac{6z(2z^3+1) - 3z^2 6z^2}{(2z^3+1)^2} = \frac{6z(1-z^3)}{(2z^3+1)^2}$$

Critical Points: 0, 1, ω, ω^2
Postcritical Points: 0, 1, ω, ω^2

Table of values:

z	∞	0	1	$\frac{-1}{2}$	ω	$\frac{-\omega}{2}$	ω^2	$\frac{-\omega^2}{2}$
$f(z)$	0	0	1	1	ω^2	ω^2	ω	ω

f is decreasing on the intervals $(-\infty, 0)$ and $(1, \infty)$
f is increasing on the interval $(0, 1)$

We conclude that f maps $[-\frac{1}{2}, \infty]$ to $[0, 1]$ in 3-to-1 fashion. This proves that the subdivision rule presentation of f at the top of the next page is correct.

We note that this NET map is the reciprocal of our second cubic Newton NET map.

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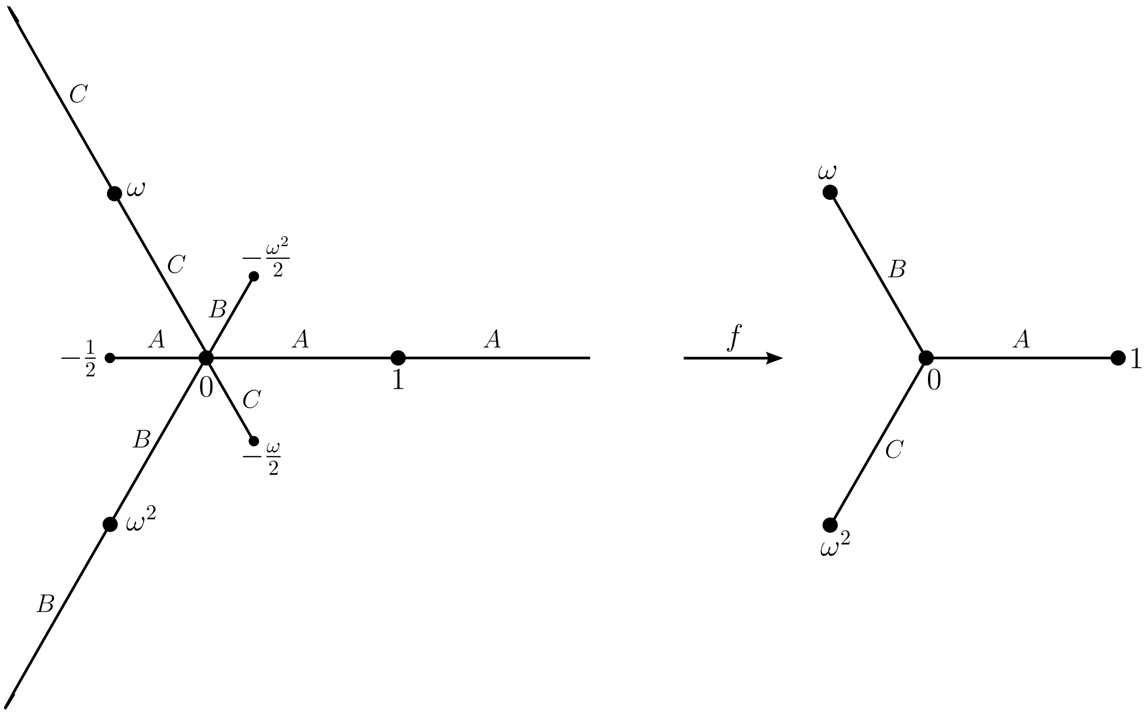

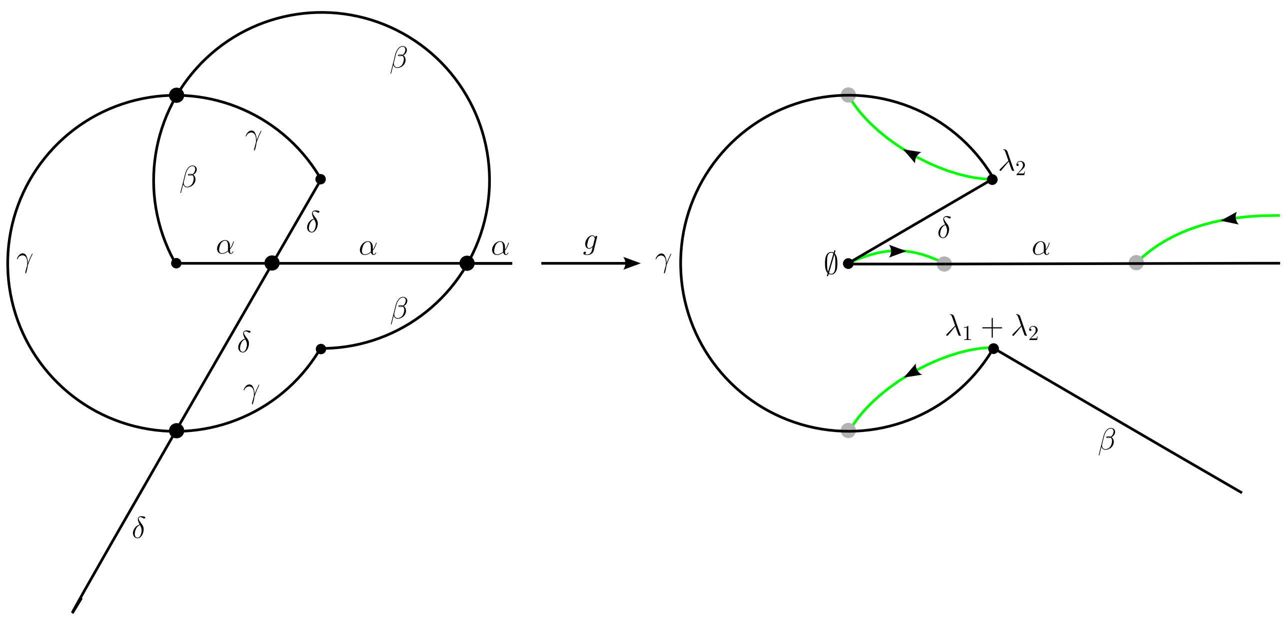

$$\frac{p}{q} = \frac{0}{1} \quad d = 3$$
$$\frac{r}{s} = \frac{3}{-1} \quad e = 3$$

$$A = \begin{bmatrix} \frac{q}{d} & \frac{s}{e} \\ \frac{p}{d} & \frac{r}{e} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ 0 & 1 \end{bmatrix}^{-1}$$
$$= \begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix} \qquad b = \lambda_1$$