

Blacksburg Math Circle
Problems on Combinatorics 9/17/2022

1. Given a pool of 30 students, how many ways can we choose a 3-person government consisting of a president, vice-president, and treasurer?
 - (a) First assume that no one person can hold more than one office.
 - (b) What if the same person can hold many offices?
2. We toss a coin ten times. How many different sequences of heads and tails can we obtain?
3. How many ways are there to fill in a Special Sport Lotto card? In this lotto you must predict the results of 13 hockey games, indicating either a victory for one of two teams, or a draw.
4. How many ways are there to put eight rooks on a chessboard so that they do not attack each other?
5. There are 7 Virginia Tech students and 6 William and Mary students sitting in a row of 13 seats in Lane Stadium. How many ways can this be done if
 - (a) there are no restrictions on how they sit?
 - (b) all the Virginia Tech students sit next to each other and all William and Mary students sit next to each other?
 - (c) each student has only neighbors from the opposite school?
6. How many ways can you choose a team from 11 people where the team must have at least one person and must have a designated captain?
7. There are 10 Virginia Tech students and 10 William and Mary students that have volunteered to help with an event. They have to be paired into teams of two. In how many ways can they be paired off if
 - (a) each team has to have 1 student from Virginia Tech and 1 student from William and Mary?
 - (b) there are no restrictions on how the teams are chosen other than each person can only be on one team?
8. How many diagonals are there in a convex n -gon?
9. How many even 3-digit numbers have no repeating digits?

10. The Hermetian alphabet consists of only three letters: A, B and C. A word in this language is an arbitrary sequence of no more than four letters. How many words does the Hermetian language contain?
11. How many digits are used to enumerate the pages in a book with 250 pages? (First page is labeled with 1 and last page is labeled with 250.)
12. An n -bit string is an n -digit binary number, i.e. a string of just zeros and ones. How many 10-bit strings contain *exactly* 5 consecutive zeroes (no more, no less)? For example, we would not count 0000000111 (too many consecutive zeros), but we would count 1110000011 and 0011000001.
13. Three different flavors of pie are available, and seven children are each given a slice of pie in such a way that at least two children get different flavors. How many ways can this be done?
14. How many six-digit numbers have at least one even digit?
15. How many four-digit numbers with no digits repeated using only the digits 0, 1, 2, 3, 4, 5 or 6 are there? How many of them are even?
16. Find the number of positive integers n such that $0 < n^2 < 1000$.
17. How many subsets of the set $\{1, 2, 3, \dots, 30\}$ have the property that the sum of their elements is greater than 232?