

Math Circle, Nov 19

Maziar Ouliaei-Nia

Problems for Congruences, Chinese Remainder Theorem, and Wilson's Theorem

1. Find the multiplicative inverse of every number from 1 to 22 (mod 23). Do all these numbers have multiplicative inverses?
2. What is $22!$ (22 factorial) (mod 23)?
3. Do all the numbers from 1 to 23 have multiplicative inverses (mod 24)?
4. Compare the results of problem 1 and problem 3 and explain the difference and why it happens.
5. Solve the congruence $6x \equiv 11 \pmod{17}$.
6. Find the smallest positive integer that satisfies all the following congruences (Chinese Remainder Theorem):
 $x \equiv 2 \pmod{3}$ $x \equiv 2 \pmod{4}$ $x \equiv 1 \pmod{5}$