

## Test Version: A

**INSTRUCTIONS:** Please enter your **NAME**, **ID Number**, **Test Version**, and your **CRN** on the opscan sheet. The CRN should be written in the field labeled **Class ID** and the test version (A, B, or C) under **Test ID**. Leave the Date, Instructor/Class, Test Name, and Time fields blank. Darken the appropriate circles below your ID number, below Class ID, and beside Test Version. **Use a number 2 pencil.** Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1-14 of the opscan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the opscan sheet with your answers and the question sheets at the end of this part of the final exam.

**Exam Policies:** You may not use a book, notes, formula sheet, calculator or a computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: \_\_\_\_\_

Name (printed): \_\_\_\_\_

Student ID #: \_\_\_\_\_

[1] Let  $f(x, y, z) = x + y + z$ . The rate of change of  $f$  at point  $P(1, 1, 2)$  in the direction of point  $Q(3, 0, 4)$  is

A) 1

B) 3

C) 4

D) 7

[2] The traces (cross-sections)  $x = k$  of the surface  $x^2 - 2x - y^2 = z^2$  are circles for

A) All  $k$ .

B) All  $k \neq 0$ .

C)  $k > 0$  only.

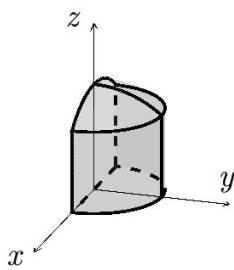
D)  $k < 0$  and  $k > 2$  only.

[3] A table of values for a differentiable function  $f(x, y)$  is shown below. Estimate  $\frac{\partial f}{\partial y}(2, 4)$ .

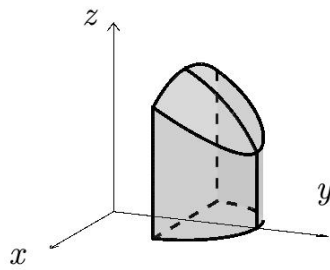
	$x = 2$	$x = 4$	$x = 6$
$y = 2$	11	15	19
$y = 4$	9	11	13
$y = 6$	7	7	7

- A)  $-2$                       B)  $-1$                       C)  $1$                       D)  $2$

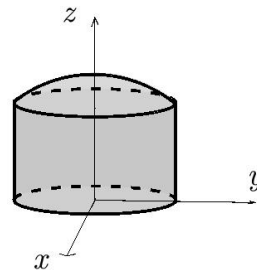
[4]  $\int_0^\pi \int_0^{2\sin\theta} \int_0^{\sqrt{9-r^2}} r \, dz \, dr \, d\theta$  represents the volume of which of the following solids?



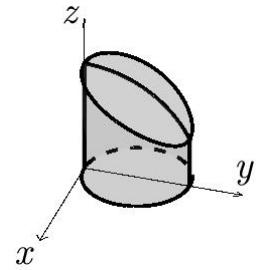
A)



B)



C)



D)

[5] Let  $\mathbf{u}$  and  $\mathbf{v}$  be three-dimensional vectors and  $f(x, y, z)$  a scalar function. Which of the following expressions is **not** meaningful?

- A)  $(\mathbf{u} \cdot \mathbf{v}) \times \nabla f$       B)  $(\mathbf{u} \times \mathbf{v}) \cdot \nabla f$       C)  $(\mathbf{u} \times \mathbf{v}) \times \nabla f$       D)  $\mathbf{u} \cdot \mathbf{v} + |\nabla f|$

[6] Approximate  $\int_{-3}^1 \int_0^4 xy^2 \, dy \, dx$  using the midpoint rule with  $m = 1$  and  $n = 2$  (i.e. divide the region into two equal subregions by drawing a horizontal line segment).

- A)  $-64$                       B)  $64$                       C)  $-80$                       D)  $80$

[7] Which vector is normal to the plane containing the points  $(0, -1, 3)$ ,  $(1, -2, 5)$  and  $(-1, 2, -2)$ ?

- A)  $\langle 3, 9, 3 \rangle$                       B)  $\langle 1, -3, 1 \rangle$                       C)  $\langle 2, -6, -4 \rangle$                       D)  $\langle -3, -9, 6 \rangle$

[8] A particle moves over a period of time according to the position function  $\mathbf{r}(t) = \langle 2t^{3/2}, 2, 3t \rangle$  from the position  $\langle 0, 2, 0 \rangle$  to  $\langle 2, 2, 3 \rangle$ . What is the distance traveled by the particle?

- A)  $\frac{3}{2}\sqrt{6}$                       B)  $2\sqrt{2}$                       C)  $\sqrt{13}$                       D)  $4\sqrt{2} - 2$

[9] Let  $E$  be the part of the solid that is bounded by  $z = 2 - x^2 - y^2$  and  $z = 6 - 2x^2 - 2y^2$  and also satisfies  $x \geq 0$  and  $y \geq 0$ . Which of the following triple integrals represents the volume of  $E$ ?

- A)  $\int_0^{\sqrt{2}} \int_0^{\sqrt{2-x^2}} \int_{2-x^2-y^2}^{6-2x^2-2y^2} dz \, dy \, dx$                       B)  $\int_0^{\sqrt{3}} \int_0^{\sqrt{3-x^2}} \int_{2-x^2-y^2}^{6-2x^2-2y^2} dz \, dy \, dx$   
C)  $\int_{\sqrt{2}}^{\sqrt{3}} \int_{\sqrt{2-x^2}}^{\sqrt{3-x^2}} \int_{2-x^2-y^2}^{6-2x^2-2y^2} dz \, dy \, dx$                       D)  $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{2-x^2-y^2}^{6-2x^2-2y^2} dz \, dy \, dx$

[10] Let  $S$  be the surface  $x^2 = 1 + z^2$  and  $P = (4, 0, 0)$ . Which of the following statements are true?

- 1: The closest point to  $P$  on  $S$  has distance 3 from  $P$ .
- 2: The farthest point to  $P$  on  $S$  has distance 5 from  $P$ .

- A) Only 1.                      B) Only 2.                      C) Both 1 and 2.                      D) Neither 1 nor 2.

[11] Let  $P$  be the plane tangent to the surface  $z = e^{-x} \cos y$  at the point where  $x = 0$  and  $y = 0$ . Which point lies on plane  $P$ ?

- A)  $(1, 0, 2)$                       B)  $(1, 0, 1)$                       C)  $(1, 0, 0)$                       D)  $(0, 1, 0)$

[12] A particle initially moves on a rail for  $0 \leq t \leq 1$ , goes off the rail at  $t = 1$ , and falls with a constant downward acceleration  $\mathbf{a} = \langle 0, 0, -2 \rangle$  for  $t > 1$ . The position of the particle for  $0 \leq t \leq 1$  is given by  $\mathbf{r}(t) = \langle t, t^2, t^2 \rangle$ . Find the velocity of the particle at  $t = 2$ .

- A)  $\langle 1, 4, 4 \rangle$                       B)  $\langle 1, 2, 0 \rangle$                       C)  $\langle 0, 0, -4 \rangle$                       D)  $\langle 1, 0, -4 \rangle$

[13]

$(x, y)$	$f(x, y)$	$f_x(x, y)$	$f_y(x, y)$	$f_{xx}(x, y)$	$f_{yy}(x, y)$	$f_{xy}(x, y)$
$(2, 1)$	2	0	1	-1	-3	-1
$(2, 0)$	-1	0	0	2	2	3
$(0, 0)$	2	0	0	2	2	1

Let  $f(x, y)$  be a function with continuous second partial derivatives. The table above shows for the function  $f$

- A) no local maximum and one saddle point.  
 B) a local maximum at one point and no saddle point.  
 C) a local maximum at one point and one saddle point.  
 D) a local maximum at two points and no saddle point.

[14] Let  $D$  be the region inside the parallelogram with vertices  $(0, 0)$ ,  $(2, 0)$ ,  $(3, 1)$ , and  $(1, 1)$ .

Compute  $\iint_D y^3 \, dA$ .

- A)  $\frac{1}{4}$                       B)  $\frac{1}{2}$                       C)  $\frac{3}{4}$                       D)  $\frac{9}{20}$