Instructions:

• Fill in A, B or C in the Test Version section.

• Enter your NAME, ID Number, CRN (under Class ID) and write A, B, or C (under Test ID) on the op-scan sheet.

• Darken the appropriate circles below your ID number and Class ID (CRN). Use a number 2 pencil. Machine grading may ignore faintly marked circles.

• Mark your answers to the test questions in rows 1–14 of the op-scan sheet. Your score on this test will be the number of correct answers.

• You have one hour to complete this portion of the exam. Turn in the op-scan sheet with your answers, this exam, and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, calculator, or a computer.

Name (printed): _________________________________

Student ID #: _________________________________

Honor Pledge: I have neither given nor received unauthorized assistance on this exam.

Signature: _________________________________
1. Let \( f \) be a continuous function such that
\[
\int_{1}^{x} (t + 1) f(t) \, dt = x^4
\]
What is \( f(3) \)?

(A) \( f(3) = 108 \)

(B) \( f(3) = 27 \)

(C) \( f(3) = 162/25 \)

(D) \( f(3) = 81/4 \)

2. Let \( f \) and \( g \) be continuous functions, and suppose that
\[
\int_{-4}^{2} f(x) \, dx = 2a \quad \text{and} \quad \int_{-4}^{2} g(x) \, dx = 3b
\]
Find \( \int_{-4}^{2} (3f(x) + 2g(x) + a + b) \, dx \).

(A) \( 7(a + b) \)

(B) \( a + b \)

(C) \( 12(a + b) \)

(D) \( 3a + 4b \)

3. Which one of the following functions satisfies the hypotheses of the Mean Value Theorem on their respective intervals?

(A) \( f(x) = |x^2 - 4| \) on \([-3, 0]\)

(B) \( f(x) = x^{1/3} \) on \([-2, 2]\)

(C) \( f(x) = |2x - 1| \) on \([-1.5, 1.5]\)

(D) \( f(x) = (x - 1)^{1/5} \) on \([1, 6]\)
4. From a piece of wire 60 cm long, a piece is cut off and discarded. The remaining wire is bent into a rectangle with one side equal to the length of the discarded piece. If we want to maximize the area that can be enclosed by such a rectangle and $x$ represents the length (in cm) of the discarded piece, which of the following is the function that should be maximized?

(A) $A(x) = 30x - \frac{3}{2}x^2$

(B) $A(x) = \left(30 - \frac{x}{2}\right)^2$

(C) $A(x) = x\left(\frac{60 - x}{4}\right)$

(D) $A(x) = \frac{60 - x}{x} + x^2$

5. The linearization of $f(x) = \ln(x) - cx$, where $c$ is an unknown constant, centered at $a = 1$ is

$L(x) = m(x - 1) + 2$

Find the value of $m$.

(A) $m = -2$

(B) $m = -1$

(C) $m = 3$

(D) $m = 2 - \frac{\ln(2)}{2}$

6. Let $g$ be a differentiable function for all real numbers. Suppose $f(x) = (x - 3)^2g(x) + \pi$. If $g(a) > 0$ and $g'(a) < 0$ for some $a < 3$, then

(A) $f''(a) > 0$

(B) $f''(a) = 0$

(C) $f''(a) < 0$

(D) Not enough information is given to make a conclusion about $f''(a)$
7. A particle moves with position function \( s(t) = -\frac{16}{3}t^3 + 32t^2 + 64, \quad t \geq 0. \) The particle slows down when

(A) \( 0 < t < 2 \)
(B) \( 2 < t < 4 \)
(C) \( 0 < t < 2 \) and \( 2 < t < 4 \)
(D) \( t > 4 \)

8. The expression

\[
\frac{1}{10} \left( \ln \sqrt{\frac{11}{10}} + \ln \sqrt{\frac{12}{10}} + \ln \sqrt{\frac{13}{10}} + \ldots + \ln \sqrt{\frac{20}{10}} \right)
\]

is a Riemann sum approximation with \( n = 10 \) rectangles of equal width for

(A) \( \frac{1}{10} \int_{0}^{1} \ln \sqrt{x+1} \, dx \)
(B) \( \int_{0}^{1} \ln \sqrt{x+1} \, dx \)
(C) \( \frac{1}{10} \int_{0}^{1} \ln \sqrt{\frac{x+1}{10}} \, dx \)
(D) \( \int_{0}^{1} \ln \sqrt{\frac{x+1}{10}} \, dx \)

9. Evaluate the definite integral \( \int_{-\pi/2}^{\pi/2} \cos(x) \sin^2(x) \, dx. \)

(A) 0 \hspace{2cm} (B) \( \frac{\pi^3}{3} \) \hspace{2cm} (C) \( \frac{2}{3} \) \hspace{2cm} (D) \( \frac{1}{3} \)
10. The graph of \( y = \sqrt{2x+1} \) is given below. Find the largest possible value of \( \delta \) such that if \( |x - 4| < \delta \), then \( |\sqrt{2x+1} - 3| < \frac{1}{2} \).

\[
\begin{array}{c|c|c|c|c|c}
\hline
x & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
y & 2.5 & 3 & 3.5 & & & \\
\hline
\end{array}
\]

(A) \( \delta = \frac{11}{8} \) \\
(B) \( \delta = \frac{13}{8} \) \\
(C) \( \delta = \frac{21}{8} \) \\
(D) \( \delta = \frac{45}{8} \)

11. Which of the following are correct applications of limit laws?

I. \( \lim_{x \to 0^+} \frac{e^x - 1}{e^{2x} - 1} = \lim_{x \to 0^+} \frac{e^x - 1}{e^{2x} - 1} \)

II. \( \lim_{x \to \infty} \frac{5x^2 + 10x}{3x^2 + 6x - 8} = \lim_{x \to \infty} \frac{5x^2 + 10x}{3x^2 + 6x - 8} \)

(A) Only I  \\
(B) Only II  \\
(C) Both I and II  \\
(D) None of the above

12. Let \( f(x) = 2^{1-x} \). Find \( f^{(5)}(3) \).

(A) \( f^{(5)}(3) = -\frac{(\ln(2))^3}{2^4} \)  \\
(B) \( f^{(5)}(3) = -\frac{\ln(3)}{2^2} \)  \\
(C) \( f^{(5)}(3) = -\frac{1}{2^2(\ln(2))^5} \)  \\
(D) \( f^{(5)}(3) = -\frac{(\ln(2))^5}{2^2} \)
13. Suppose we have a continuous function, $f$, such that $f(a) < 3$ and $f(b) > 6$ for some real numbers $a$ and $b$ with $a < b$. Then, there must exist a real number $c$ in $(a, b)$ such that

(A) $f(c) = 4.25$
(B) $f(c) = 0$
(C) $f'(c) = 4.25$
(D) $f'(c) = 0$

14. The graph of a function $f$ is given below. Assume that the curve outside of this view is as indicated by the arrows.

The curve $y = f(x)$ has

(A) 1 horizontal asymptote and 1 vertical asymptote.
(B) 1 horizontal asymptote and 2 vertical asymptotes.
(C) 2 horizontal asymptotes and 1 vertical asymptote.
(D) 2 horizontal asymptotes and 2 vertical asymptotes.