Instructions: Fill in A, B or C in the Test Version section. Then enter your NAME, ID Number, CRN (under Class ID) and write A, B, or C (under Test ID) on the op-scan sheet. Darken the appropriate circles below your ID number and Class ID (CRN). Use a number 2 pencil. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1–15 of the op-scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op-scan sheet with your answers, this exam and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: __________________________________________

Name (printed): ______________________________________

Student ID #: ________________________________________

1. The integral $\int \frac{1}{\sqrt{x^2 - 1}} \, dx$ can be rewritten as which of the following?

   (A) $\int \sec(\theta) \, d\theta$  
   (B) $\int 1 \, d\theta$  
   (C) $\int \cot(\theta) \, d\theta$  
   (D) $\int \frac{1}{\cos(\theta) \sin(\theta)} \, d\theta$

2. Evaluate $\int \frac{1}{x^3 + x} \, dx$.

   (A) $\ln |x| - \frac{1}{2} \ln |x^2 + 1| + C$  
   (B) $\frac{1}{2} \ln |x + 1| + \frac{1}{2} \ln |x - 1| - \ln |x| + C$

   (C) $\ln |x| + C$  
   (D) $\ln |x| - \frac{1}{2x^2} + C$
3. Find the volume of the region graphed on the interval \([\frac{\pi}{4}, \frac{3\pi}{4}]\) below when it is rotated around \(y = 2\).

(A)  \(\pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (2 - \cos(x))^2 - (2 - \sin(x))^2 \, dx\)

(B)  \(\pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (x - 2)(\sin(x) - \cos(x)) \, dx\)

(C)  \(\pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (2^2 - (\sin(x) - \cos(x))^2) \, dx\)

(D)  \(\pi \int_{-1}^{1} (y - 2)(\arcsin(y) - \arccos(y)) \, dy\)

4. A cylindrical tank with height 7 meters and radius 5 meters is filled to a height of 4 meters with water, which has a density of 1000 kg/m³. Set up an integral to find the amount of work required to pump the water out of a spout at the top of the cylindrical tank.

(A)  \(9800 \int_0^4 25\pi (7 - y) \, dy\)

(B)  \(9800 \int_0^4 \pi y^2 (7 - y) \, dy\)

(C)  \(9800 \int_0^7 25\pi (4 - y) \, dy\)

(D)  \(9800 \int_4^7 25\pi (3 - y) \, dy\)
5. Set up an integral to find the moment about the x-axis, \( M_x \), of the region bounded by \( y = 2 - x^2 \) and \( y = x \) with density \( \rho = 3 \).

\[
(A) \quad \int_{-2}^{1} \frac{3}{2} \left( (2 - x^2)^2 - x^2 \right) \, dx \\
(B) \quad \int_{-2}^{1} 3 \left( (2 - x^2) - x \right) \, dx \\
(C) \quad \int_{-2}^{1} 3 \left( (2 - x^2) - x \right) \cdot x \, dx \\
(D) \quad \int_{-2}^{1} \frac{3}{2} \left( (2 - x^2) - x \right)^2 \, dx
\]

6. Find the average value of \( f(x) = x \sin(x) \) on \([0, \pi]\).

\[
(A) \quad 1 \quad (B) \quad \frac{\pi}{2} \quad (C) \quad \pi \quad (D) \quad \pi^2 - 4
\]

7. Let \( \sum_{n=1}^{\infty} a_n \) be a series such that \( \lim_{n \to \infty} a_n = 0 \) and \( a_n > 0 \). Let \( s_k \) denote the \( k \)th partial sum, \( s_k = \sum_{n=1}^{k} a_n \). Which of the following statements could be true about \( s_k \)?

\[
(A) \quad \lim_{k \to \infty} s_k = 6. \\
(B) \quad \lim_{k \to \infty} s_k = 0. \\
(C) \quad s_k < 10 \text{ for all } k, \text{ but } \sum_{n=1}^{\infty} a_n \text{ diverges.} \\
(D) \quad \lim_{k \to \infty} s_k = -3.
\]
8. Suppose a function $f(x)$ has the following Maclaurin series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x - 1)^n}{n!}.$$

Which of the following statements is true?

(A) $f'(x) = f(x)$ for all $x$.
(B) $f''(1) = \frac{1}{2!}$.
(C) $f'(1) = 0$.
(D) $f(1) = 0$.

9. Which of the following series converges conditionally?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n) + 1}$

(B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n}$
10. Let \( r = 2 - 2 \sin \theta \) and \( r = 2 \sin \theta \) be polar curves given in the graph below. To find the area of the shaded region, we would use

\[
\begin{align*}
\text{(A)} & \quad \frac{1}{2} \int_{0}^{\pi/6} (2 \sin \theta)^2 \, d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} (2 - 2 \sin \theta)^2 \, d\theta \\
\text{(B)} & \quad \frac{1}{2} \int_{0}^{\pi/6} (2 - 2 \sin \theta)^2 - (2 \sin \theta)^2 \, d\theta \\
\text{(C)} & \quad \frac{1}{2} \int_{0}^{\pi/2} (2 - 2 \sin \theta)^2 + (2 \sin \theta)^2 \, d\theta \\
\text{(D)} & \quad \frac{1}{2} \int_{0}^{\pi/6} 2 \sin \theta \, d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} 2 - 2 \sin \theta \, d\theta
\end{align*}
\]

11. Use the trapezoidal rule with \( n = 3 \) to approximate \( \ln(2) \).

(Recall that: \( \ln(x) = \int_{1}^{x} \frac{1}{t} \, dt \)).

\[
\begin{align*}
\text{(A)} & \quad \frac{7}{10} \\
\text{(B)} & \quad \frac{3}{2} \\
\text{(C)} & \quad \frac{7}{8} \\
\text{(D)} & \quad \frac{5}{21}
\end{align*}
\]

12. The open interval of convergence of the power series

\[
\sum_{n=0}^{\infty} \frac{(2x - 4)^n}{3^n n^3}
\]

is

\[
\begin{align*}
\text{(A)} & \quad x \in \left( \frac{1}{2}, \frac{7}{2} \right) \\
\text{(B)} & \quad x \in (-3, 3) \\
\text{(C)} & \quad x \in (-\infty, \infty) \\
\text{(D)} & \quad x \in \left( -\frac{3}{2}, \frac{3}{2} \right)
\end{align*}
\]

13. Which of the given series converges to the function \( f(x) = \frac{2}{1 + 9x^2} \) on the appropriate interval of convergence.

\[
\begin{align*}
\text{(A)} & \quad \sum_{n=0}^{\infty} 2(-1)^n (3x)^{2n} \\
\text{(B)} & \quad \sum_{n=0}^{\infty} \frac{2x^{2n}}{3^n} \\
\text{(C)} & \quad \sum_{n=0}^{\infty} 2(3x)^{2n} \\
\text{(D)} & \quad \sum_{n=0}^{\infty} \frac{2(-1)^n}{(3x)^{2n}}
\end{align*}
\]
14. Evaluate \[ \lim_{x \to 1^+} \frac{1}{x-1} - \frac{1}{\ln(x)} \]

(A) \(-\frac{1}{2}\)  (B) 0  (C) \(\infty\)  (D) \(e^{-1/2}\)

15. Evaluate \[ \int_{-1}^{1} \frac{e^x}{e^x - 1} \, dx \]

(A) Diverges  (B) \(\ln\left|\frac{e - 1}{1/e - 1}\right|\)  (C) \(\ln\left|\frac{e - 1}{1/e - 1}\right| - 1\)  (D) 1

16. Two students are told that the lifetime \(T\) (in years) of an electric component is given by the exponential probability density function \(f(t) = e^{-t}\) \(t \geq 0\). They are asked to find time \(t = L\) where a typical component is 60% likely to exceed. The two students had the following approaches:

Student A: \(0.6 = \int_{L}^{\infty} e^{-t} \, dt\)

Student B: \(0.6 = 1 - \int_{0}^{L} e^{-t} \, dt\)

Which student has the correct setup to be able to find \(L\)?

(A) Both  
(B) Student A  
(C) Student B  
(D) Neither Student